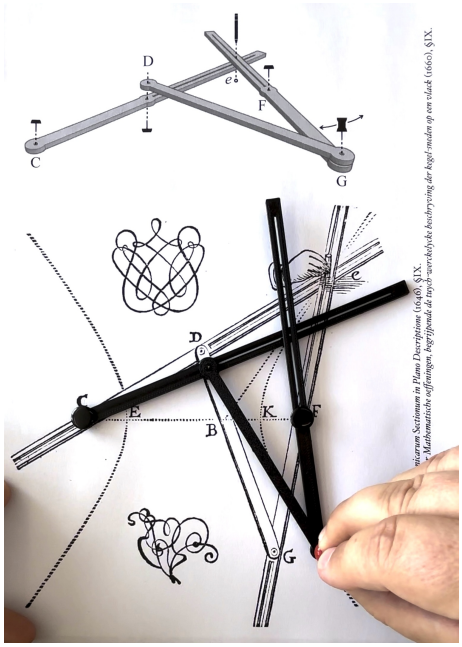
“Alle problemen van de meetkunde”: de hoogmoedige visie van Descartes

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https://github.com/viktorblasjo/conics-instruments



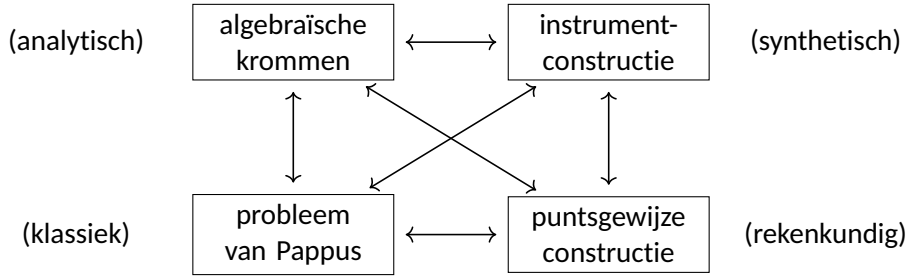
LA

GEOMETRIE.

LIVRE PREMIER.

Des probleſmes qu'on peut conſtruire ſans y employer que des cercles & des lignes droites.

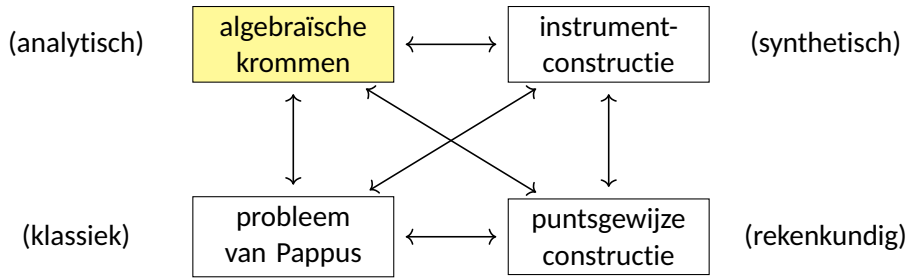
TOus les Probleſmes de Geometrie ſe peuuent facilement reduire a tels termes, qu'il n'eſt beſoin par aprés que de connoiſtre la longeur de quelques lignes droites, pour les conſtruire.



Christiaan Huygens:

"door het fraaie verband van zijn uiteenzettingen"

— Andriesse, Titan, 465; "en la belle liaison de ses expositions" `https://www.dbnl.org/tekst/huyg003oeuv10_01/huyg003oeuv10_01_0137.php`


(analytisch)
algebraïsche krommen
instrument-constructie
(synthetisch)
(klassiek)
probleem van Pappus
puntsgewijze constructie
(rekenkundig)

$yy \propto cy - \frac{cx}{b}y + ay - ac.$

de laquelle on connoiſt que la ligne E C eſt du premier genre, comme en effect elle n'eſt autre qu'vne Hyperbole.

$$x^4 + 4x^3 - 19xx - 106x - 120 \propto 0$$

$z \propto \frac{1}{2}a + \sqrt{\frac{1}{4}aa + bb}.$

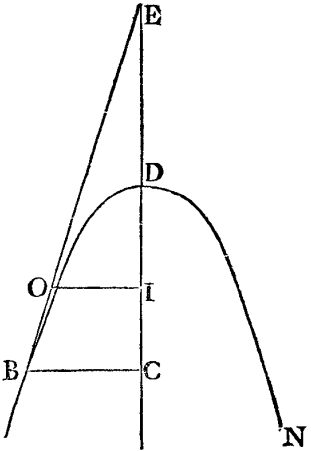
$z \propto b.$ ou

$z^2 \propto -az + bb.$ ou

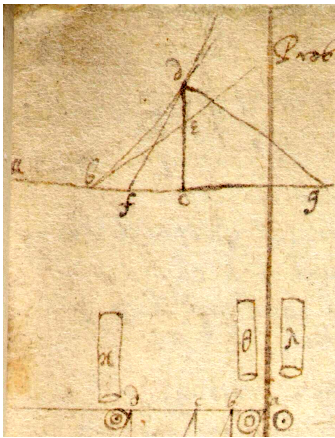
$z^3 \propto +az^2 + bbz - c^3.$ ou

$z^4 \propto az^3 - cz + d^4.$ &c.

$$\frac{xx}{ab} \qquad \frac{x^2}{ab}$$

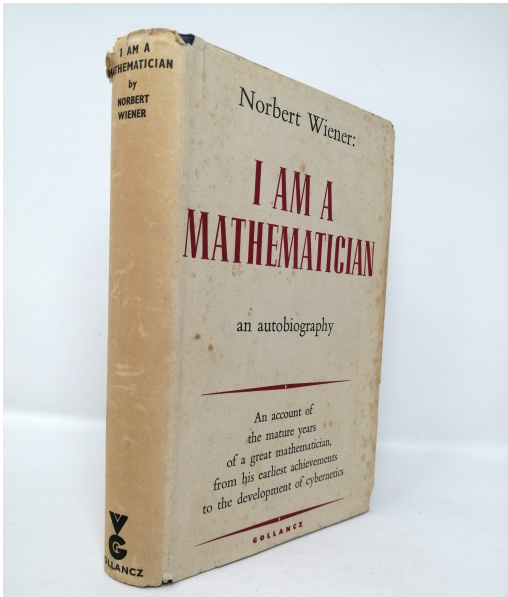


Sit data, verbi gratiâ, Parabole B D N, cujus vertex D, diameter D C, & punctum in ea datum B, ad quod ducenda eſt recta B E, tangens parabolen, & in puncto E, cum diametro concurrens, ergo ſumendo quodlibet punctum O I, in recta B E, & ab eo ducendo ordinatam O I, à puncto autem B, ordinatam B C major erit proportio C D, ad D I, quàm quadrati B C, ad quadratum O I, quia punctum O, eſt extra parabolen, ſed propter ſimilitudinem triangulorum, ut B C, quad. ad O I, quad. ita C E, quad. ad I E, quad. Major igitur erit proportio C D ad D I, quàm quadrati C E ad quad. I E, Cum autem punctum B detur, datur applicata B C, ergo punctum C datur etiam C D, Sit igitur C D, æqualis D, datæ. Ponatur C E, eſſe A, ponatur C I eſſe E, ergo D, aut D — E habebit majorem rationem, quàm A^2 ad $A^2 + E^2 - A$, in E. Et ducendo inter ſe medias & extremas D in $A^2 + D$ in $E^2 - D$ in A in E, majus erit quàm D, in $A^2 - A^2$ in E, Adæquentur igitur juxta ſuperiorem methodum, demptis itaque communibus D, in $E^2 - D$, in A in E adæquabitur $- A^2$ in E, aut quod idem eſt, D in E^2, $+ A^2$ in E, adæquabitur D in A in E, Omnia dividantur per E, ergo D in $E + A^2$ adæquabitur D in A, elidatur D in E, ergo A^2 æquabitur D in A^2, ideoque A æquabitur D, ergo C E, probavimus duplam ipſius C D, quod quidem ita ſe habet.



Problema. Of Vsury. $ab=a=$ the principle. be $\beta=\beta x$ [illegible]
$ce=\frac{dx}{\varepsilon}=$ the vse due for y^{e} principle for y^{e} time
$cd=y=$ y^{e} vse for y^{e} principle & vse du
~~as y^{e} sume or prime~~ of a tangent to y^{e} line
as y^{e} sume (ab) or principle drawne into y^{e} time
So is ~~ab+dc~~ the sume $ab+dc$ drawne into y^{e} ti
therefore $ax:\frac{dx}{\varepsilon}::a+y$ in $\frac{dx}{\varepsilon}$: y . $axy=$
$\varepsilon av=ady+dyy$. $v=\frac{ady+dyy}{\varepsilon a}$. $\partial=\varepsilon$. $\frac{ay+yy}{\varepsilon a}$

https://cudl.lib.cam.ac.uk/view/MS-ADD-04004/31



"If there is any one quality which marks **the competent mathematician** more than any other, I think it is the power to **operate with temporary emotional symbols** and to organize out of them a semipermanent, recallable language. If one is not able to do this, one is likely to find that his ideas evaporate from the sheer difficulty of preserving them in an as yet **unformulated shape**." (86)



Leibniz introduceerde de notaties $\int$ en d in een manuscript van 29 oktober 1675.

“Wat ik aan deze analyse het meest waardeer, is dat ze ons hetzelfde voordeel biedt ten opzichte van de ouden wat betreft de meetkunde van Archimedes, als wat Viète en Descartes ons boden voor de meetkunde van Euclides en Apollonius.”

videtur admirabile, et novo huic calculo magni adjumenti loco futurum, nempe quod sit

$\overline{\text{Omn. } \overline{l}^2} \,/\, 2 \sqcap \text{omn. } \overline{\overline{\text{omn. } l}\,\frac{l}{a}}$ qualiscunque sit l. id est si omnes l. ducantur in ultimam, et aliae omnes l. rursus in suam ultimam, et ita quoties id fieri potest, summa omnium aequabitur dimidiae summae quadratorum quorum latera sunt summae ipsorum l, seu omnes l. pulcherrimum et minime obvium theorema. Tale est etiam Theorema: $\text{omn. } \overline{xl} \sqcap x\ \text{omn. } \overline{l} - \text{omn. omn. } l$. ponendo l. esse terminum ~~[illegible]~~ progressionis ~~x~~ et x esse numerum qui exprimit locum seu ordinem ipsius l. ei respondentis, seu x. esse numerum ordinalem, l. rem ordinatam. Nota in his calculis observari potest lex homogeneorum. Nam si omn. praefigatur numero seu rationi, vel infinite parvo, fit linea, si lineae fit superficies, si superficiei fit corpus, et ita in infinitum circa ad dimensiones.

Utile erit scribi ∫. pro omn. ~~[illegible]~~ ut ∫l ~~[illegible]~~ pro omn. l. id est summa ipsorum l.

Itaque fiet $\frac{\overline{\int \overline{l}}^2}{2a} \sqcap \int \int \overline{l}\,\frac{l}{a}$ et $\int \overline{xl} \sqcap x\int \overline{l} - \int\int \overline{l}$. Et ita apparebit semper observari legem homogeneorum, quod utile est, ut calculi errores vitentur. Nota si analytice detur ∫l. dabitur etiam l. Ergo si detur ∫∫l dabitur etiam l. sed non si detur l. dabitur et ∫l. Semper $\int x \sqcap \frac{x^2}{2}$. Nota omnia haec theoremata vera de seriebus in quibus differentiae terminorum ad terminos rationem habent minorem quavis assignabili. $\int x^2 \sqcap \frac{x^3}{3}$.

$\int \frac{l}{2}$ etc. Nota jam si termini summandi affecti sint, quomodo hinc afficiatur summa regulam generalem habere.

v.g. $\int \frac{a}{b}$ $\int \frac{a}{b}$ $\int \frac{a}{b}\overline{l} \sqcap \frac{a}{b} \times \int \overline{l}$. scilicet si $\frac{a}{b}$ sit terminus constans ducendus est in maximum ordinalem. Quod si sit terminus inconstans ~~vel esse exprimitur relatione ordinata~~ hunc tractari non posse nisi ad ipsum l. reduci possit vel utrumque ad quantitatem communem semper ordinalem. Nota quotiescunque in aequatione tetragonistica non nisi una est litera variabilis ut l. tunc potest poni esse terminus constans, et ipsa [illegible] ∫ enim l. x. Et huic fundamento innititur theorema: ... Eodem loco modo talem innumera [illegible] possunt solvi, ut

$\frac{\int \overline{l}^2}{2} \sqcap$ ~~[illegible]~~ $\int \overline{l}$ $\int\int \overline{l}$ idest $\frac{x^2}{2} \sqcap \int x$.

$\int l$ $\int \frac{a}{a}\overline{l}^2 + ba^2 + \overline{l}^3 +$ ~~[illegible]~~ $\int l^3 \sqcap e a^3$ quaeritur qualis sit ~~[illegible]~~ e. fiet $e \sqcap \frac{cx^3}{3} + ba^2x + \frac{x^4}{4} + xa^3$

(1) (2) (3) (4)

Nimirum $\int l^3 \sqcap x$. quia $l \sqcap a$. supponitur calculi causa. (1) (2) (3) (4) $\int \overline{l}^1 \sqcap x$



uitdrukking	dimensie	soort
x	1	lijn
$4xy$	2	vlak
xxy	3	lichaam

“Er valt een wet van homogeniteit waar te nemen. Want wanneer omn wordt toegevoegd aan een getal, een verhouding of een oneindig kleine grootheid, levert dit een lijn op; wordt het toegevoegd aan een lijn, dan ontstaat een vlak; wordt het toegevoegd aan een vlak, dan ontstaat een lichaam; en zo gaat dit oneindig door wat betreft de dimensies.”

integraal	resultaat	dimensie	soort
$\int 5$	$= 5x$	1	lijn
$\int x$	$= \frac{1}{2}x^2$	2	vlak
$\int xx$	$= \frac{1}{3}x^3$	3	lichaam
$\vdots$	$\vdots$	$\vdots$	$\vdots$

$yy \propto cy - \frac{cx}{b}y + ay - ac.$

de laquelle on connoiſt que la ligne E C eſt du premier genre, comme en effect elle n'eſt autre qu'vne Hyperbole.

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$z \propto b.$ ou

$z^2 \propto -az + bb.$ ou

$z^3 \propto +az^2 + bbz - c^3.$ ou

$z^4 \propto az^3 - cz + d^4.$ &c.



"Het is nuttig om $\int$ te schrijven voor omn., zoals $\int l$ voor omn. l, dat wil zeggen de som van alle l. Dus … $\int \overline{xl} \sqcap x\overline{\int l} - \int \overline{\int l}$. Op deze manier wordt de wet van homogeniteit steeds in acht genomen, wat nuttig is om rekenfouten te voorkomen."

$$\int xf\,dx = xF - \int F\,dx$$

Een bijzonder geval van partiële integratie

$$\int gf\,dx = gF - \int g'F\,dx$$



"$\int$ staat voor som, *d* voor verschil."

"Als $\int l \sqcap ya$, stellen we $l \sqcap \frac{ya}{d}$. Want als $\int$. de dimensie vergroot, zo verlaagt *d* de dimensie."

$$\int f\,dx = F \quad \leftrightarrow \quad f = F'$$

$$\int f = ya \quad \leftrightarrow \quad f = \frac{ya}{d}$$

Videtur admirabile, et novo hoc calculo magni adjumenti loco futurum, nempe quod sit

$\frac{\text{omn. } l^2}{2} \sqcap \text{omn. } \overline{\text{omn. } l}\, \frac{l}{a}$. qualiscunque sit l. id est si omnes l. ducantur in ultimam, et aliae omnes l, rursus in suam ultimam, et ita quoties fieri potest, summa horum omnium aequabitur dimidiae summae quadratorum, quorum latera sunt summae ipsarum l, seu omnes l. pulcherrimum et minime obvium theorema. Tale est etiam Theorema: $\text{omn. } xl \sqcap x\, \overline{\text{omn. } l} - \text{omn. } \overline{\text{omn. } l}$. ponendo l. esse terminum progressionis et x esse numerum qui exprimit locum seu ordinem ipsius l. ei respondentis, seu x. esse numerum ordinalem, l. rem ordinatam. Nota in his calculis observari potest lex homogeneorum. Nam si omn. praefigatur numero seu rationi, vel infinite parvo, fit linea, si lineae fit superficies, si superficiei fit corpus, et ita in infinitum circa dimensiones.

Utile erit scribi $\int$. pro omn. ut $\int l$ pro omn. l. id est summa ipsorum l.

Itaque fiet $\frac{\overline{\int l}^2}{2a} \sqcap \int \overline{\int l}\, \frac{l}{a}$ et $\int xl \sqcap x \int l - \int\int l$. Et ita apparebit semper observari legem homogeneorum, quod utile est, ut calculi errores vitentur. Nota si analytice detur $\int l$. dabitur etiam l. Ergo si detur $\int\int l$ dabitur etiam l. sed non si detur l. dabitur et $\int l$. Semper $\int x \sqcap \frac{x^2}{2}$. Nota omnia haec theoremata vera de seriebus in quibus differentiae terminorum ad terminos rationem habent minorem quavis assignabili. $\int x^2 \sqcap \frac{x^3}{3}$.

$\int \frac{l}{2}$ [illegible] Nota jam si termini summandi affecti sint, quomodo hinc afficiatur summa regulam generalem habemus.

Ut v.g. $\int \frac{a}{b} l$ [illegible] $\int \frac{a}{b} l \sqcap \frac{a}{b} \times \int l$. Scilicet si $\frac{a}{b}$ sit terminus constans ducendus est in maximum ordinalem. quod si sit terminus inconstans ~~vel esse exprimitur relatione ordinata~~ hunc tractari non posse, nisi ad ipsam l. reduci possit vel utramque ad quantitatem communem semper ordinalem. Nota quotiescunque in aequatione tetragonistica non nisi una est littera variabilis ut l. tunc potest omni esse terminus constans, et ipse ordinalis seu x. Et huic fundamento innititur theorema: ut

$\frac{\int l^2}{2} \sqcap \int$ [illegible] $\int \int l$ idest $\frac{x^2}{2} \sqcap \int x$. Eodem vero modo talem innumera similia possunt solvi, et

$\int l$ [illegible] $\int \frac{a}{a} l^2 + ba^2 + \int l^3 + \int l^3$ [illegible] $\int l^3 \sqcap e\, a^3$ quaeritur qualis sit e. fiet $e \sqcap \frac{cx^3}{3} + ba^2x + \frac{x^4}{4} + xa^3$

(1) (2) (3) (4)

Nimirum $\int l^3 \sqcap x$. quia $l \sqcap a$. supponitur calculi causa. (1) (2) (3) (4)

$\int l \sqcap x$

$\sqrt{a^2 + b^2}$, pour tirer la racine quarrée d' $a^2 + b^2$; Et $\sqrt{\text{C.}\, a^3 - b^3 + abb}$, pour tirer la racine cubique d' $a^3 - b^3 + abb$, & ainſi des autres.

reigle de Cardan ſeroit,

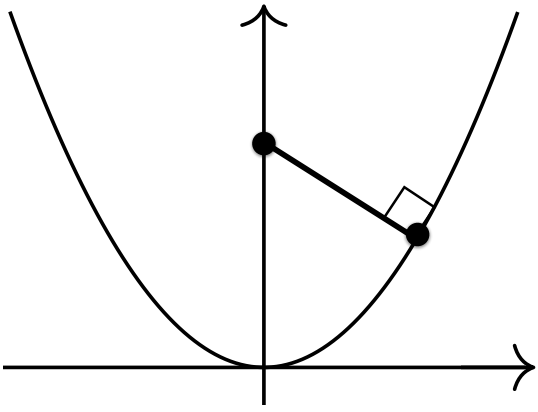
$$\sqrt{\text{C.}\, \tfrac{1}{2}q + \sqrt{\tfrac{1}{4}qq - \tfrac{1}{27}p^3}} + \sqrt{\text{C.}\, \tfrac{1}{2}q - \sqrt{\tfrac{1}{4}qq - \tfrac{1}{27}p^3}}.$$

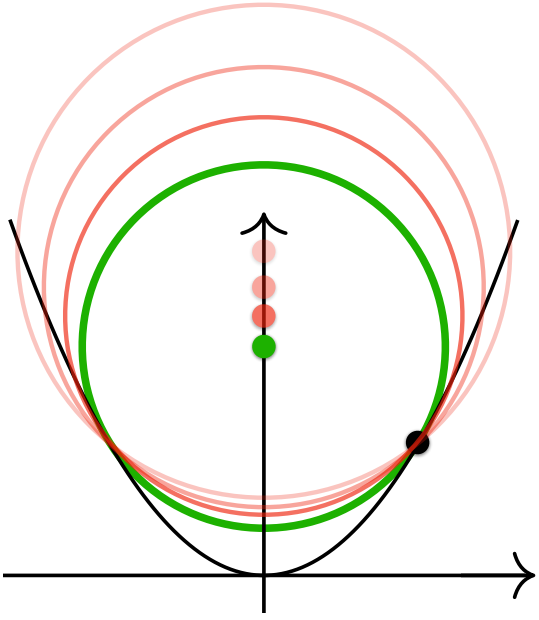
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ANNO M DC LXXXIV

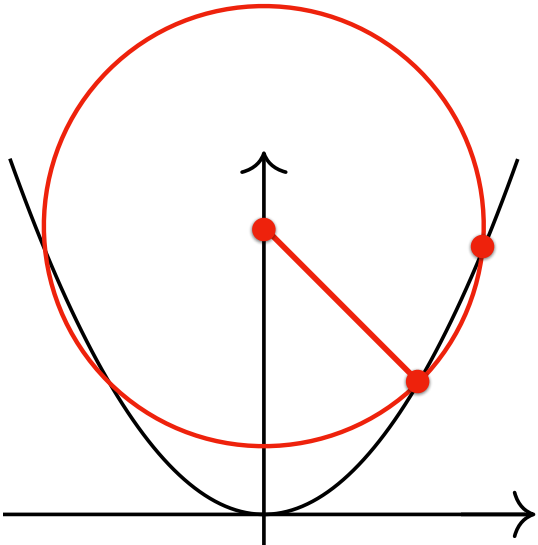
NOVA METHODUS PRO MAXIMIS ET MINIMIS, itemque tangentibus, quæ nec fractas, nec irrationales quantitates moratur, & singulare pro illis calculi genus, per G. G. L.

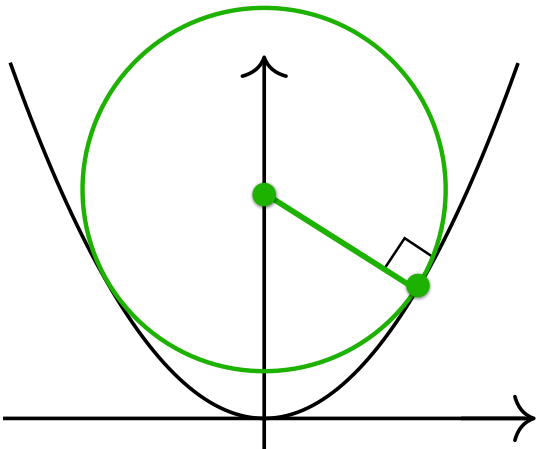
TAB. XII.

Sit axis AX, & curvæ plures, ut VV, WW, YY, ZZ, quarum ordinatæ, ad axem normales, VX, WX, YX, ZX, quæ vocentur respective, *v*, *w*, *y*, *z*; & ipsa AX abscissa ab axe, vocetur *x*. Tangentes sint VB, WC, YD, ZE axi occurrentes respective in punctis B, C, D, E.









numeris. Hae scri-
ciones radicem, ~~calculum abbreviant~~, multum abbreviantur
per hoc Theorema.

$$\overline{P+PQ}\Big|^{\frac{m}{n}} = P^{\frac{m}{n}} + \frac{m}{n}AQ + \frac{m-n}{2n}BQ + \frac{m-2n}{3n}(Q + \frac{m-3n}{4n}DQ + \&c$$

Vbi $P+PQ$ significat quantitatem cujus radix vel dimensio quaevis
vel radix dimensionis investiganda est, primum terminum quan-
titatis ejus, Q reliquos terminos divisos per primum, & $\frac{m}{n}$ nume
ralem indicem dimensionis ipsius $P+PQ$ sive dimensio illa integra
sit, sive (ut ita loquar) fracta, sive affirmativa sive negativa.
Nam sicut Analystae pro aa, aaa &c scribunt a^2 a^3, sic ego pro
$\sqrt{a}$, $\sqrt{a^3}$, $\sqrt{C}$. as &c scribo $a^{\frac{1}{2}}$, $a^{\frac{3}{2}}$, $a^{\frac{5}{3}}$, & pro $\frac{1}{a}$, $\frac{1}{aa}$, $\frac{1}{a^3}$ scribo
a^{-1}, a^{-2}, a^{-3}. Et sic pro $\frac{aa}{\sqrt{C:a^3+bbx}}$ scribo $aa\overline{\times a^3+bbx}\Big|^{-\frac{1}{3}}$, &
pro $\frac{aab}{\sqrt{C:a^3+bbx}\times a^3+bbx}$ scribo $aab\overline{\times a^3+bbx}\Big|^{-\frac{2}{3}}$: in quo ultimo casu
si $\overline{a^3+bbx}\Big|^{-\frac{2}{3}}$ concipiatur esse $\overline{P+PQ}\Big|^{\frac{m}{n}}$ in Regula; erit $P=a^3$,

$(1+x)^q$ als machtreeks: binomiaalreeks

$$\begin{aligned}
f(x) &= (1+x)^q \\
f'(x) &= q(1+x)^{q-1} \\
f''(x) &= q(q-1)(1+x)^{q-2} \\
f'''(x) &= q(q-1)(q-2)(1+x)^{q-3} \\
f''''(x) &= q(q-1)(q-2)(q-3)(1+x)^{q-4} \\
f'''''(x) &= q(q-1)(q-2)(q-3)(q-4)(1+x)^{q-5}
\end{aligned}$$

$$\begin{aligned}
f(0) &= 1 \\
f'(0) &= q \\
f''(0) &= q(q-1) \\
f'''(0) &= q(q-1)(q-2) \\
f''''(0) &= q(q-1)(q-2)(q-3) \\
f'''''(0) &= q(q-1)(q-2)(q-3)(q-4)
\end{aligned}$$

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f''''(0)}{4!}x^4 + \frac{f'''''(0)}{5!}x^5 + \cdots$$

$$(1+x)^q = 1 + qx + \frac{q(q-1)}{2!}x^2 + \frac{q(q-1)(q-2)}{3!}x^3 + \frac{q(q-1)(q-2)(q-3)}{4!}x^4 + \frac{q(q-1)(q-2)(q-3)(q-4)}{5!}x^5 + \cdots$$

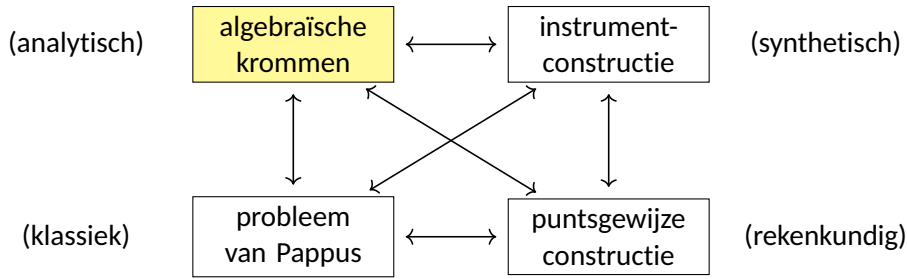
$$\underbrace{\left(x^5\right)' \qquad \left(\frac{1}{x^2}\right)' \qquad \left(\sqrt{x}\right)'}_{(x^n)'=nx^{n-1}}$$

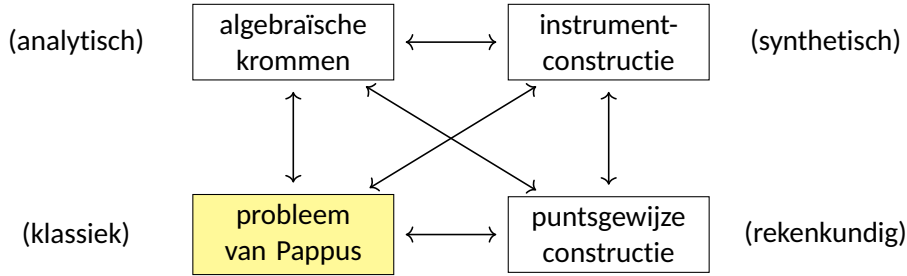
Potentiæ $d X^a$, æqu. $a \cdot X^{a-1} dx$, exempli gratia d, X^3 æqu. $3X^2 dx$

$d \frac{1}{X^a}$ æqu. $-\frac{a\,dx}{X^{a+1}}$ ex gr. si vv sit æqu. $\frac{1}{X^3}$ fiet $d\,vv$ æqu. $-\frac{3\,dx}{X^4}$.

Radices: $d, \sqrt[b]{X^a}$, æqu. $\frac{a}{b}\,dx \sqrt[b]{X^{a-b}}$ (Hinc $d \sqrt[2]{y}$ æqu. $\frac{1}{2}\,\frac{dy}{\sqrt[2]{y}}$

nam eo casu a est 1, & b est 2; ergo $\frac{1}{2} \sqrt[2]{X^{1-2}}$ est $\frac{1}{2} Y^{-1}$ jam Y^{-1} idem est quod $\frac{1}{y}$ ex natura exponentium progressionis Geometricæ, & $\sqrt[2]{y}$ est $\frac{1}{2} Y$) $d \frac{1}{\sqrt[b]{X^a}}$ æqu. $\frac{-b\,dx}{a \sqrt[b]{X^{b-a}}}$. Suffeciſſet autem regula potentiæ integræ tam ad fractas quam ad radices determinandas, potentia enim fit fracta cum exponens est negativus, & mutatur in radicem cum exponens est fractus, sed malui consequentias istas ipse deducere, quam aliis deducendas relinquere, cum sint admodum generales, & crebro occurrentes, & in re per se implicita præstet facilitati consulere.


(analytisch)
algebraïsche krommen
instrument-constructie
(synthetisch)
(klassiek)
probleem van Pappus
puntsgewijze constructie
(rekenkundig)



Descartes:

"een vraagstuk dat noch door Euclides noch door Apollonius noch door wie ook volledig is opgelost"

"Het vraagstuk waarvan de oplossing door Euclides ter hand was genomen en door Apollonius verder was uitgewerkt, zonder dat iemand het tot een goed einde had gebracht."

TABLE

Des matieres de la

GEOMETRIE.

Liure Premier.

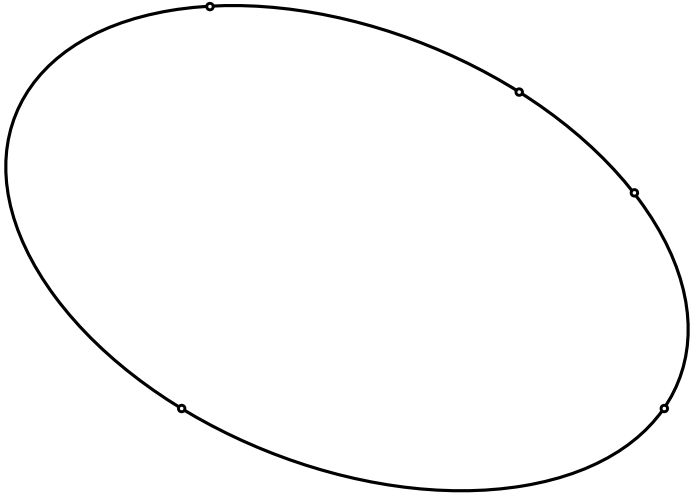
DES PROBLESMES QU'ON PEUT construire sans y employer que des cercles & des lignes droites.

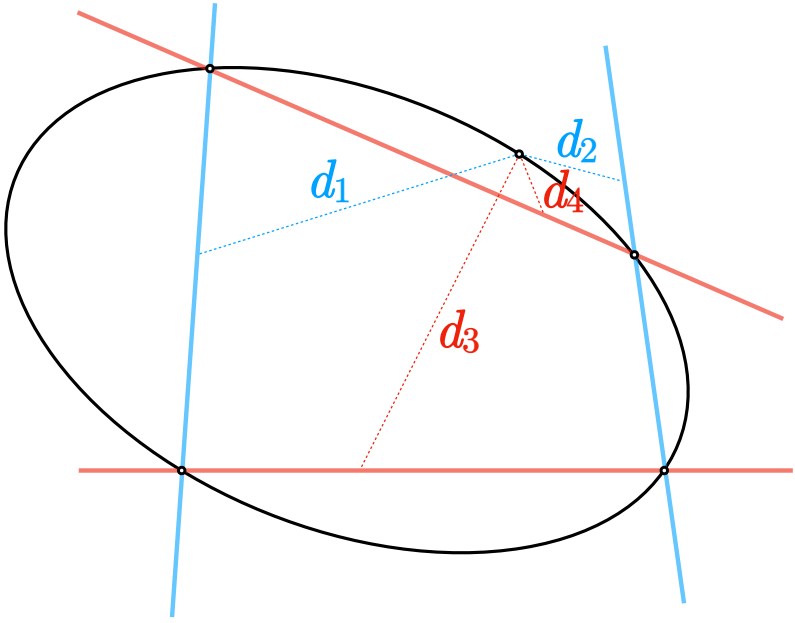
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Comment on doit poser les termes pour venir a l'Equation en cet exeple. 310

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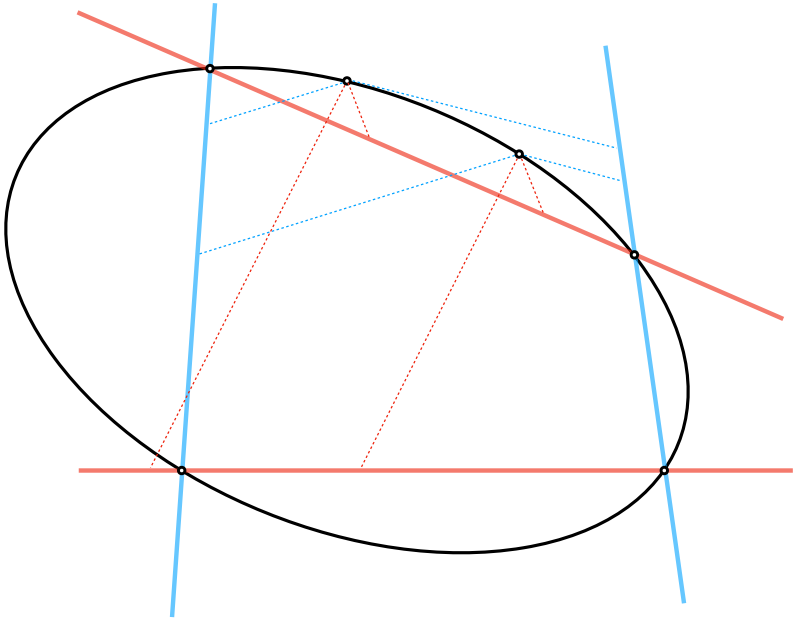
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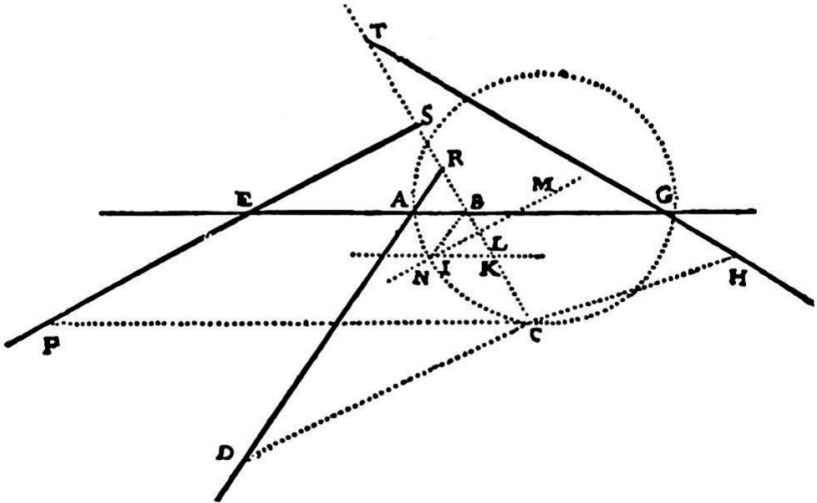


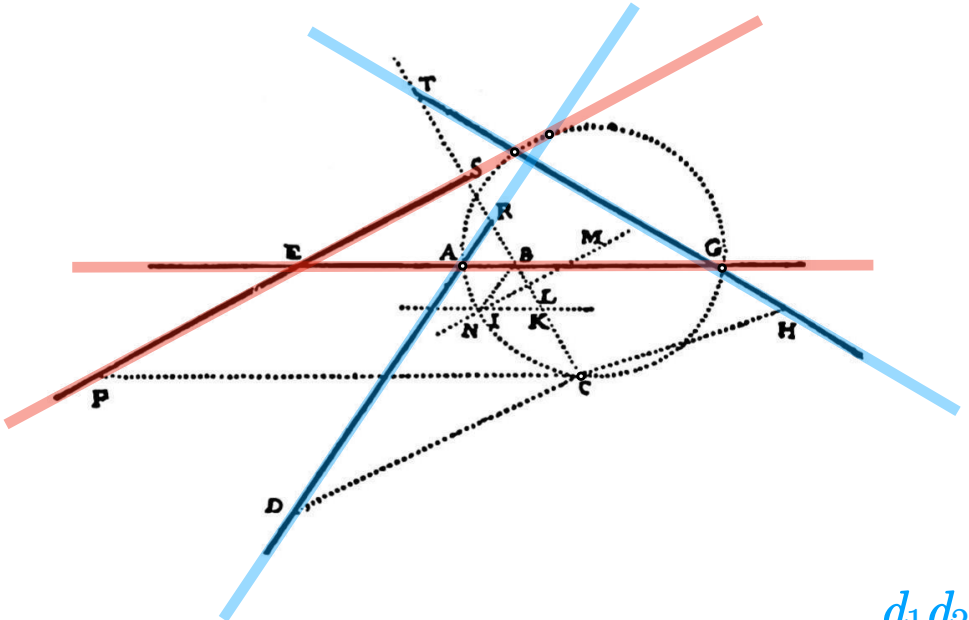


$$d_1 d_2 = \lambda d_3 d_4$$

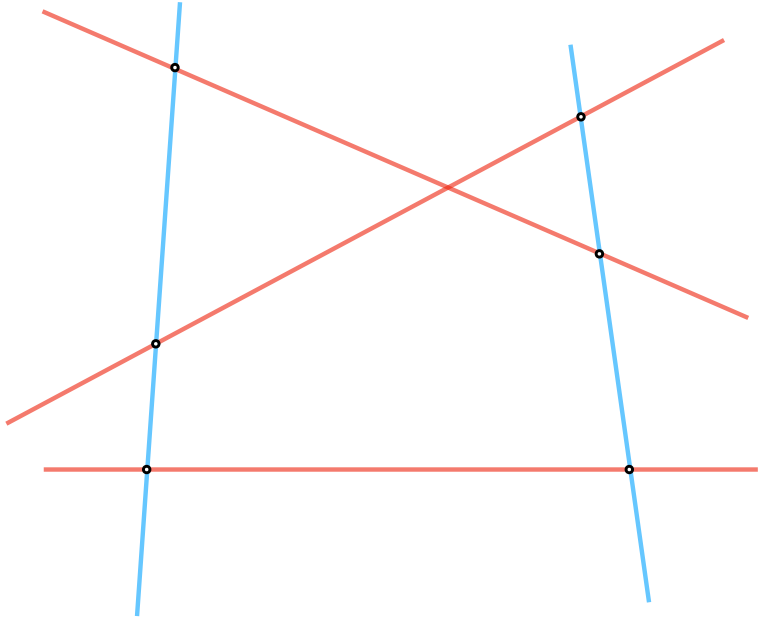


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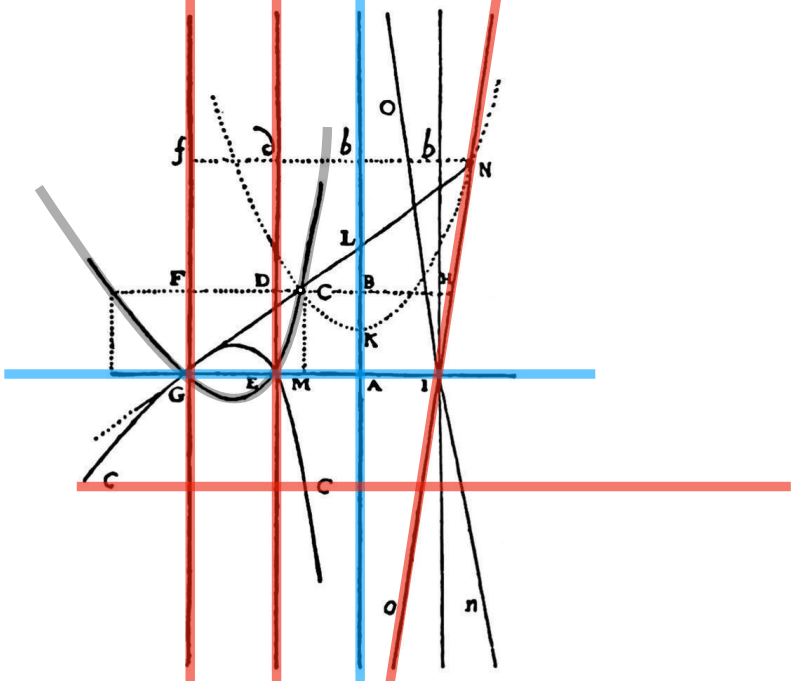

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$$d_1 d_2 = \lambda d_3 d_4$$



$$\lambda d_1 d_2 = d_3 d_4 d_5$$



$$\lambda d_1 d_2 = d_3 d_4 d_5$$

Descartes:

locus van	Euclid	Apollonius	pre-Pappus
4 lijnen	⯪	⯪	⯪
5+ lijnen	⯪	⯪	⯪

Pappus:

locus van	Euclid	Apollonius	pre-Pappus
4 lijnen	⯪	★	
5+ lijnen			⯪

Apollonius:

locus van	Euclid	Apollonius
4 lijnen	⯪	★

BIBLIOTHEEK

DESCARTES



Over de methode

Essays

Boom

Sources
in the History of Mathematics and
Physical Sciences 8

PAPPUS OF
ALEXANDRIA

BOOK 7
OF THE *COLLECTION*

PART 1. INTRODUCTION,
TEXT, AND TRANSLATION

Edited
With Translation and Commentary by

ALEXANDER JONES

Springer Science+Business Media, LLC

Quem autem dicit (Apollonius) in tertio libro locum ad

Deze meetkundige plaats met betrekking tot drie en vier lijnen, zegt hij (Apollonius) in het derde boek, is echter

(33) Thus Apollonius. The locus on three and four lines that he says, in (his account of) the third (book), was not completed by Euclid, neither he

Deze meetkundige plaats met betrekking tot drie en vier lijnen, zegt hij (Apollonius) in het derde boek, is echter niet door Euclides opgelost, noch kon hij het zelf oplossen, noch iemand anders, en alleen met wat er tot Euclides' tijd over kegelsneden was aangetoond, konden ze niet het geringste toevoegen aan wat Euclides heeft geschreven, etc.

(33) Thus Apollonius. The locus on three and four lines that he says, in (his account of) the third (book), was not completed by Euclid, neither he nor anyone else would have been capable of; no, he could not have added the slightest thing to what was written by Euclid, using only the conics that had been proved up to Euclid's time, as he himself confesses when he says that it is impossible to complete it without what he was forced to write first. **(34)** But either Euclid, out of respect for Aristaeus as meritorious for the conics he had published already, did not anticipate him, or, because he did not desire to commit to writing the same matter as he (Aristaeus), — for he was the fairest of men, and kindly to everyone who was the slightest bit able to augment knowledge, as one should be, and he was not at all belligerent, and though exacting, not boastful, the way this man (Apollonius) was, — he wrote (only) as far as it was possible to demonstrate the locus by means of the other's *Conics*, without saying that the demonstration was complete. For had he done so, one would have had to convict him, but as things stand, not at all. And in any case, (Apollonius) himself is not castigated for leaving most things incomplete in his *Conics*.

Euclides van Alexandrië

Apollonius van Perga

Pappos van Alexandrië

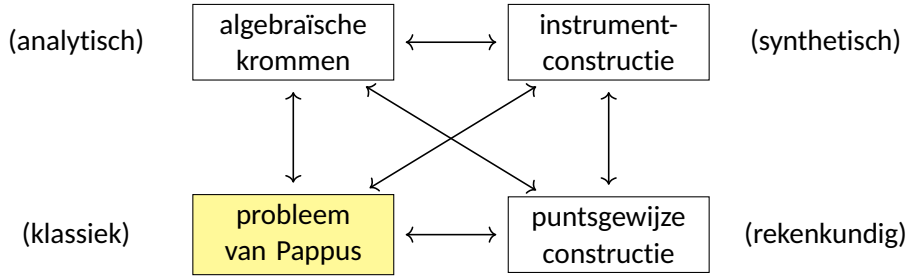
Deze meetkundige plaats met betrekking tot drie en vier lijnen, zegt hij (Apollonius) ~~in~~ over het derde boek, is echter niet door Euclides opgelost, noch kon hij het zelf oplossen, noch iemand anders, ~~en~~ alleen met wat er tot Euclides' tijd over kegelsneden was aangetoond, konden ze niet het geringste toevoegen aan wat Euclides heeft geschreven, etc.

Stanford Encyclopedia of Philosophy

Descartes' Mathematics

First published Mon Nov 28, 2011; substantive revision Mon Apr 7, 2025

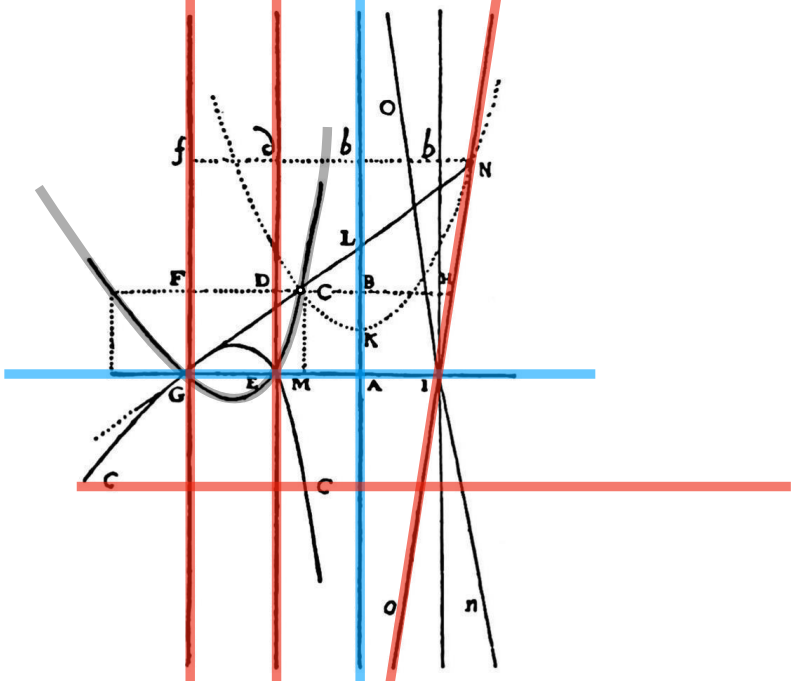
In the *Collection*, Pappus presents a solution to the three and four line versions of the problem (i.e., the versions of the problem in which we begin with three or four given lines and angles) as well as Apollonius's solution to the six-line case, which relies on his theory of conics and the transformation of areas to construct the locus of points (Pappus, 118–123). However, Pappus does not treat the general (n-line) case, and this is the advance of the solution Descartes achieves in 1632, a solution published in *La Géométrie*, where he claims that, unlike the ancients, he has found a method to successfully "determine, describe, [and] explain the nature of the line required when the question [of the Pappus Problem] involves a greater number of lines" (G, 22). And as Descartes



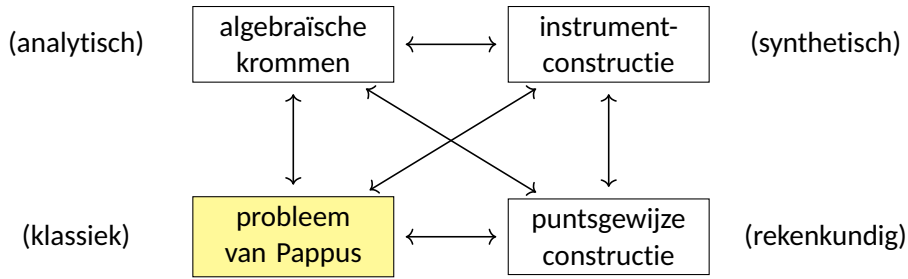
Christiaan Huygens:

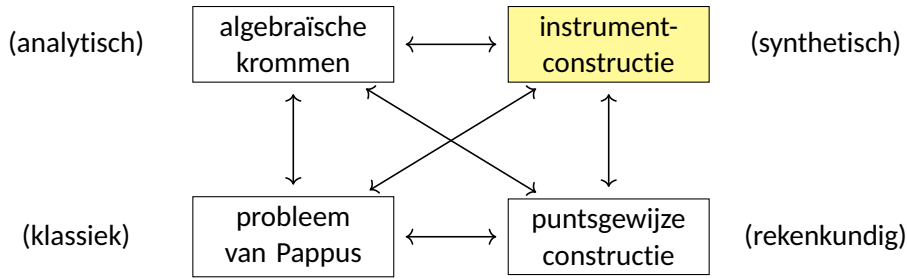
> *"Descartes schreef zo dat men zijn gissingen en verzinsels voor waar hield. En hij bereikte bij de lezers van zijn Filosofische Principes hetzelfde als bij de lezers van genoeglijke romans die over ware geschiedenissen lijken te gaan."*

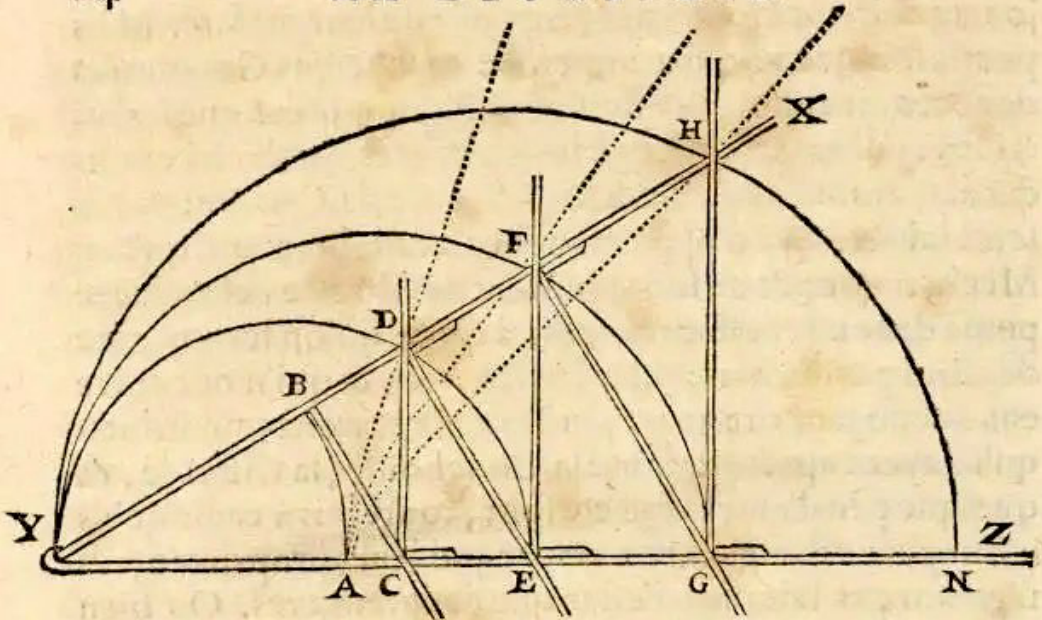
— Andriesse, Titan, 462; `https://www.dbnl.org/tekst/huyg003oeuv10_01/huyg003oeuv10_01_0137.php`

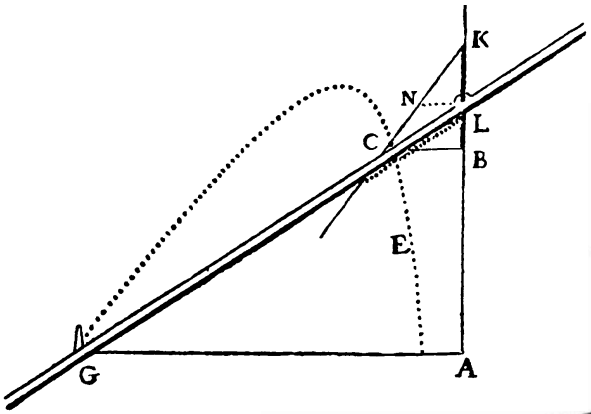


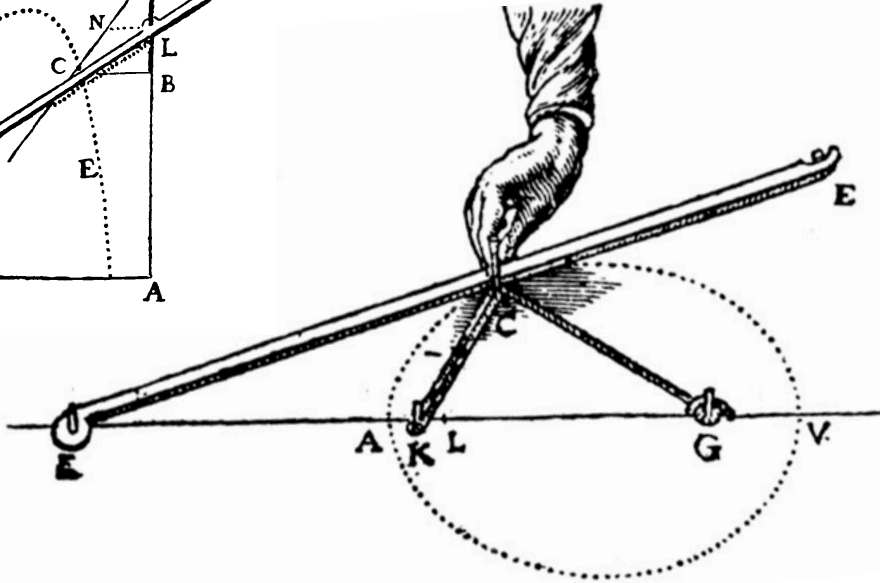
$$\lambda d_1 d_2 = d_3 d_4 d_5$$


(analytisch)
algebraïsche krommen
instrument-constructie
(synthetisch)
(klassiek)
probleem van Pappus
puntsgewijze constructie
(rekenkundig)

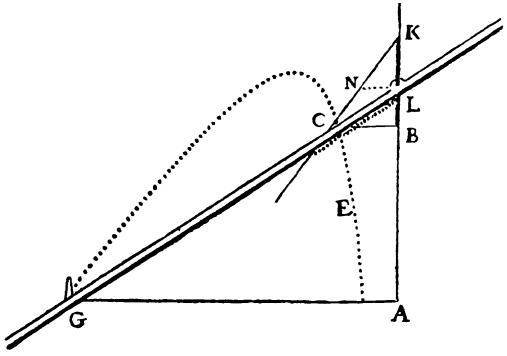

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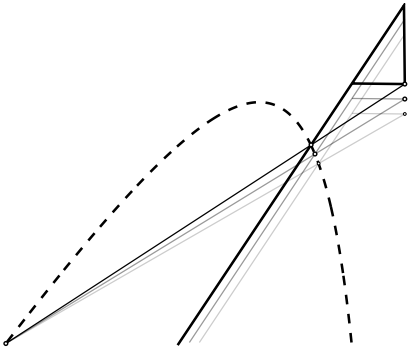

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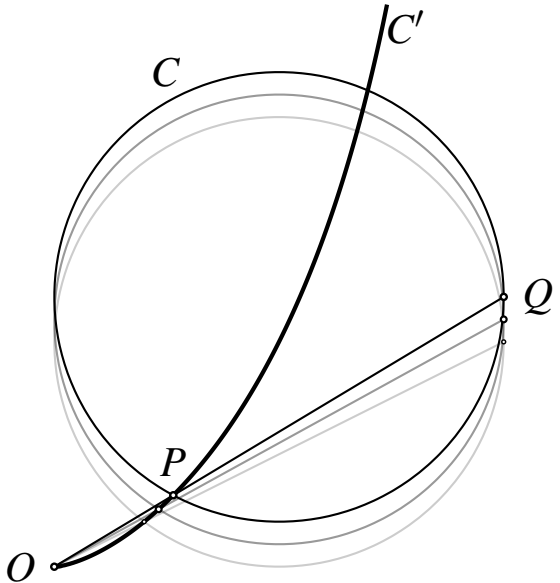

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“Draaiende liniaal en schuivende kromme”: Hyperbool

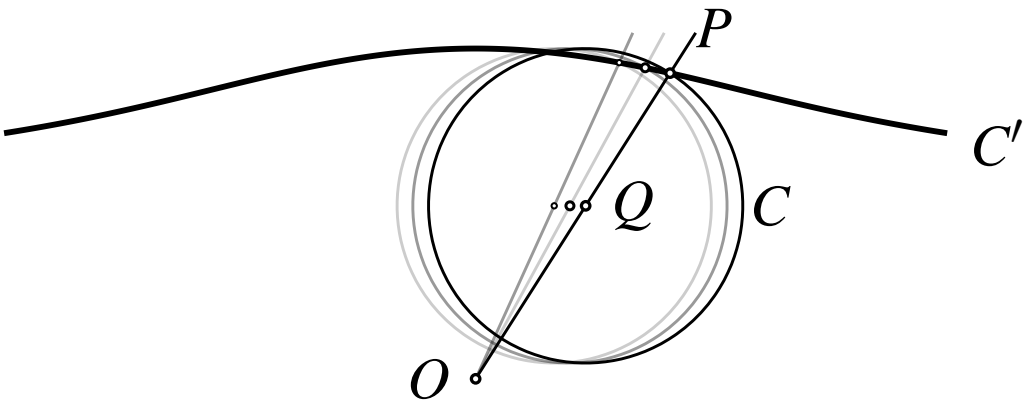




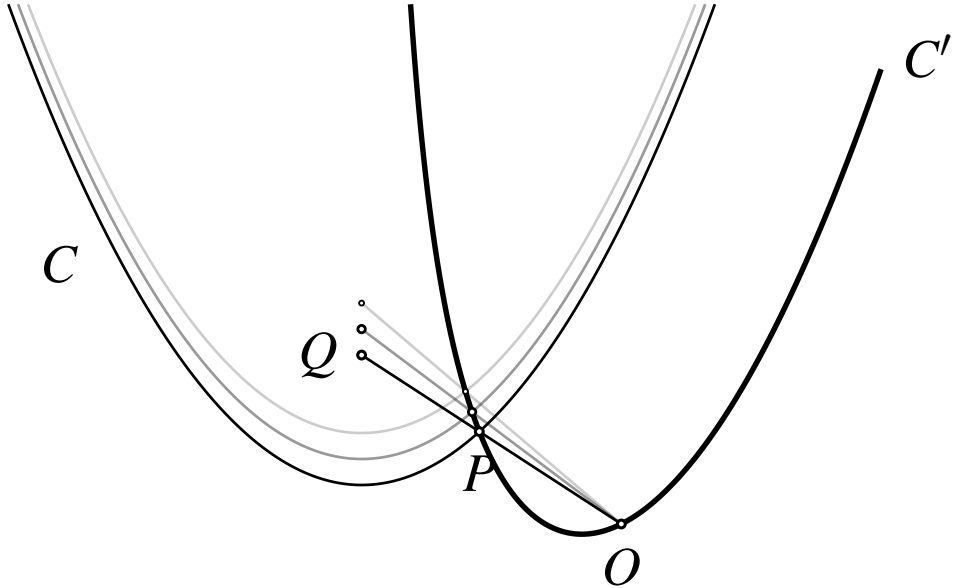
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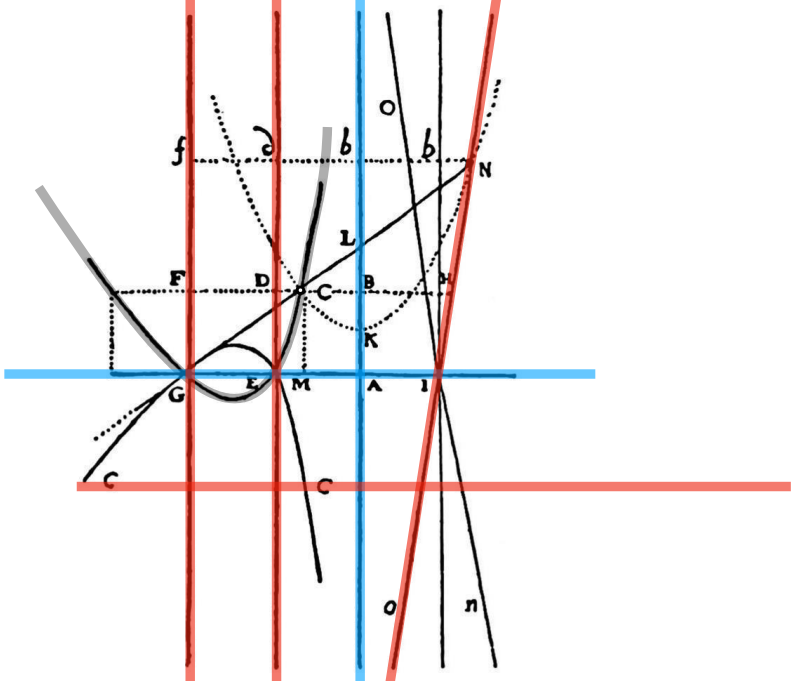


“Draaiende liniaal en schuivende kromme”: conchoïde

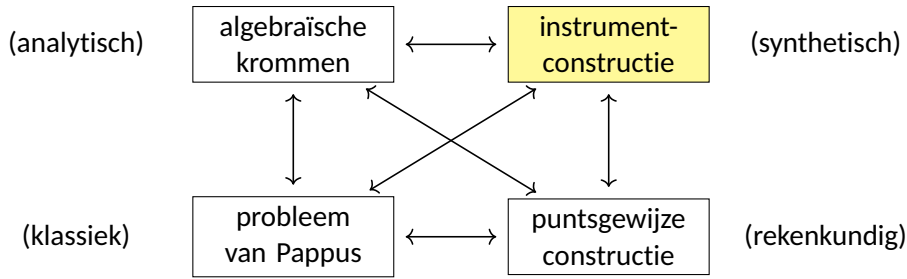


“Draaiende liniaal en schuivende kromme”: “Parabool van Descartes”



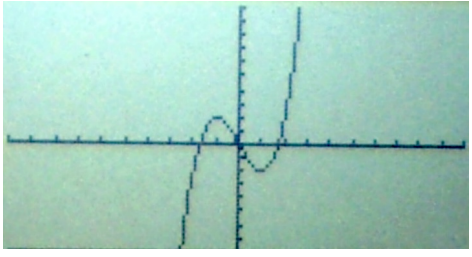


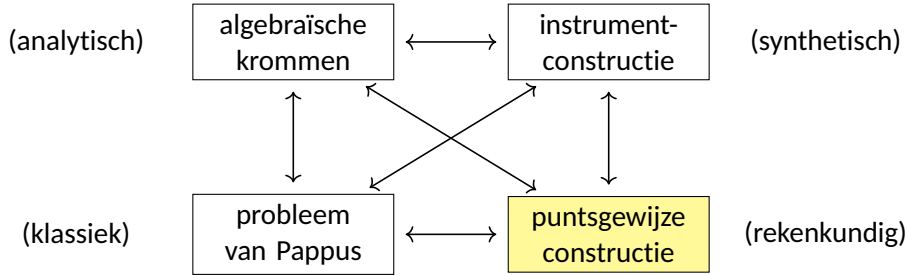
$$\lambda d_1 d_2 = d_3 d_4 d_5$$


(analytisch)
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(analytisch) algebraïsche krommen ⟷ instrument-constructie (synthetisch)

(klassiek) probleem van Pappus ⟷ puntsgewijze constructie (rekenkundig)

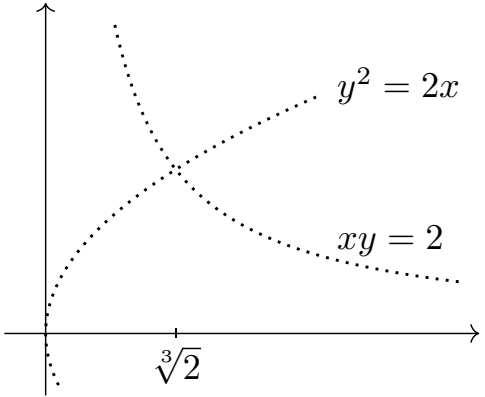




Descartes:

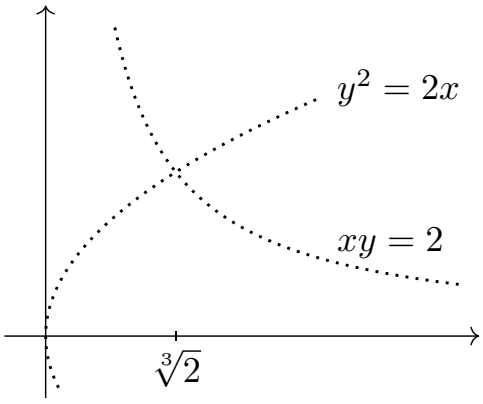
> *"Wat de lijnen voor de andere gevallen betreft, … Nu ik heb uitgelegd op welke manier oneindig veel punten kunnen worden gevonden waar de krommen doorheen gaan, meen ik ook voldoende duidelijk te hebben gemaakt hoe die krommen kunnen worden beschreven."*

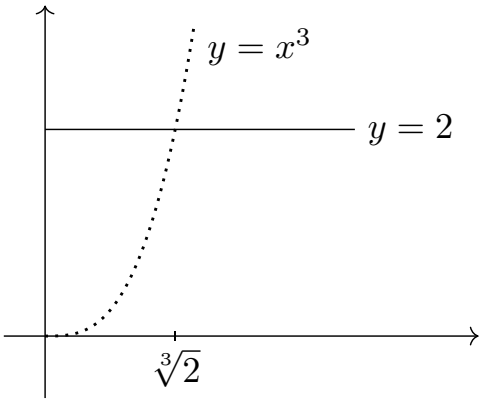
Tegen puntsgewijze constructies



“Doch soo en kan men niet seggen dat het beschrijven van een kromme linie door gevonden puncten geometrisch ofte volkomen sij, of dat sulcke beschreven linien konnen dienen tot geometrische constructie van eenighe problemata, dewijl hiertoe, nae mijn opinie, **geen kromme linien en konnen dienen als die door eenigh instrument vervolgens beschreven konnen worden**, gelijck den Cirkel door een passer; en de Conische Sectien, Conchoides en andere door de instrumenten daertoe geinventeert. Want de linien met de handt van punt tot punt getrocken alleenlijck de gesochte quantiteyt ten naesten bij konnen geven en dienvolgens niet naer de Geometrische perfectie. **Want wat helpt het sooveel puncten te vinden als men wil, indien men dat eene punct dat gesocht werdt niet en vindt?**” — Huygens (1687)

Zinloos om kegelsneden te gebruiken als puntsgewijze constructies zijn toegestaan





Proposition 1

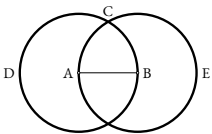
To construct an equilateral triangle on a given finite straight line.

Let AB be the given finite straight line.

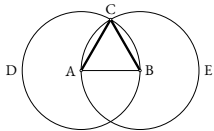


So it is required to construct an equilateral triangle on the straight line AB.

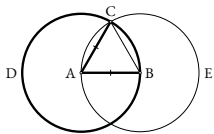
Let the circle BCD with center A and radius AB have been drawn [Post. 3], and again let the circle ACE with center B and radius BA have been drawn [Post. 3].



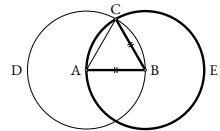
And let the straight lines CA and CB have been joined from the point C, where the circles cut one another, to the points A and B respectively [Post. 1].



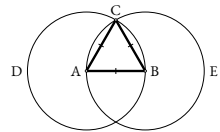
And since the point A is the center of the circle CDB, AC is equal to AB [Def. 15].



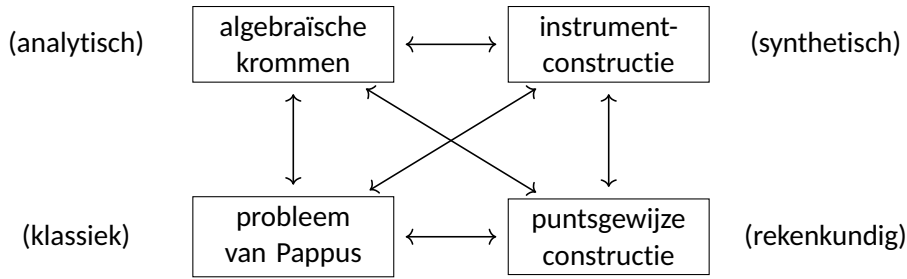
Again, since the point B is the center of the circle CAE, BC is equal to BA [Def. 15].



But CA was also shown to be equal to AB. Thus, CA and CB are each equal to AB. But things equal to the same thing are also equal to one another [C.N. 1]. Thus, CA is also equal to CB. Thus, the three straight lines CA, AB, and BC are equal to one another.



Thus, the triangle ABC is equilateral, and has been constructed on the given finite straight line AB. Which is the very thing it was required to do.


(analytisch)
algebraïsche krommen
instrument-constructie
(synthetisch)
(klassiek)
probleem van Pappus
puntsgewijze constructie
(rekenkundig)

Bedankt!

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