

Revised National Standards for Mathematics: The Story – New Challenges – Illustrative Examples

Bärbel Barzel, University of Duisburg-Essen



Pictures from IQB



Offen im Denken

The story

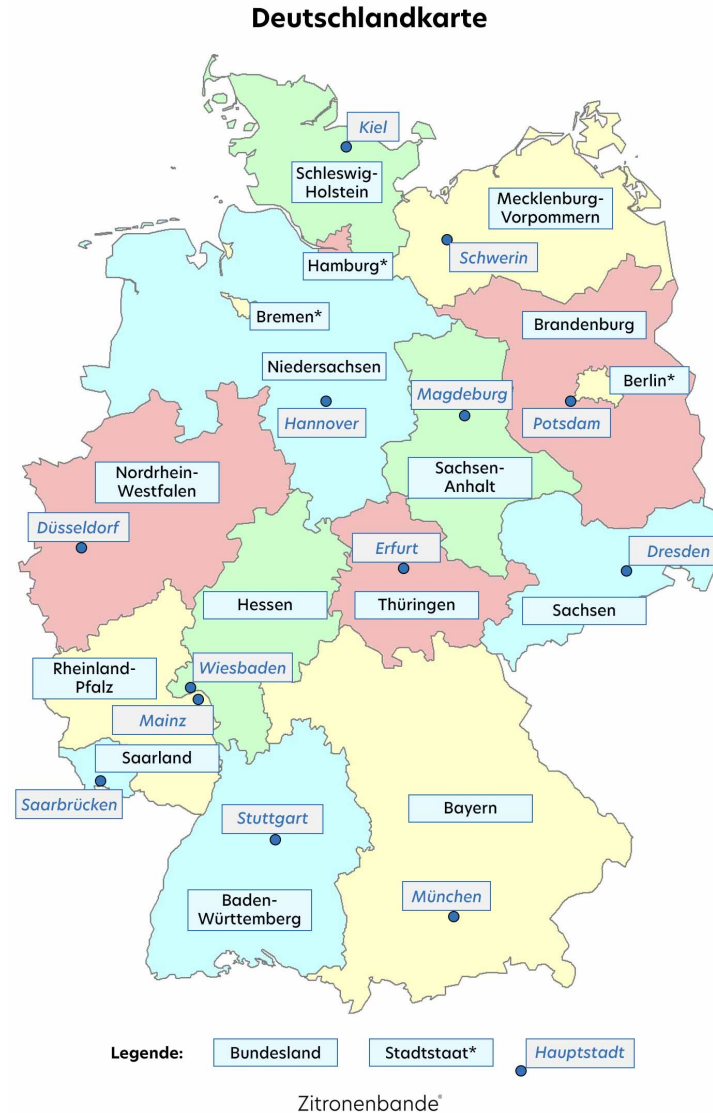
Each federal state is responsible for cultural and educational matters

BUT:
We have



(Standing Conference of the Ministers of Education & Cultural Affairs of the Länder)

„We are diverse in Germany“



1997 „Konstanzer Resolution“
Germany participates in international comparisons studies (e.g. PISA, TIMSS)

Input-focused approach



Output-focused approach

2003 National Standards

Mathematics, German, English

The story

2020 KMK decided revision of the National Standards (Mathematics, German, English) on the basis of a needs analysis, (result: medium to high need for revision)



Institut zur Qualitätsentwicklung
im Bildungswesen

Goals resulting from needs analysis
(from research and administration)

- More **Coherence over the years**
- Integration of a **taxonomy of competencies for mathematical processes**
- **Concretisation** of all competencies
- Integrate a better concretization of **education in the digital era**

Ensure as much
continuity as possible!



2016



2021

The story

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2016



2021

The story: Goals resulting from needs analysis

Coherence over the years – Core Ideas/ Content

Core Ideas from 2003 (Sec I):

- Number
- Measuring
- Space and Shape
- Functional dependency
- Data and Chance

2022

Primary (1-4)	Secondary I (5 – 9/10)	Secondary II (10 - 12/13)
Number and Operation	Number and Operation	Algorithm and Number
Quantities and Measuring	Quantities and Measuring	Measuring
Patterns, Structures and Functional dependency	Structures and Functional dependency	Functional dependency
Space and Shape	Space and Shape	Space and Shape
Data and Chance	Data and Chance	Data and Chance

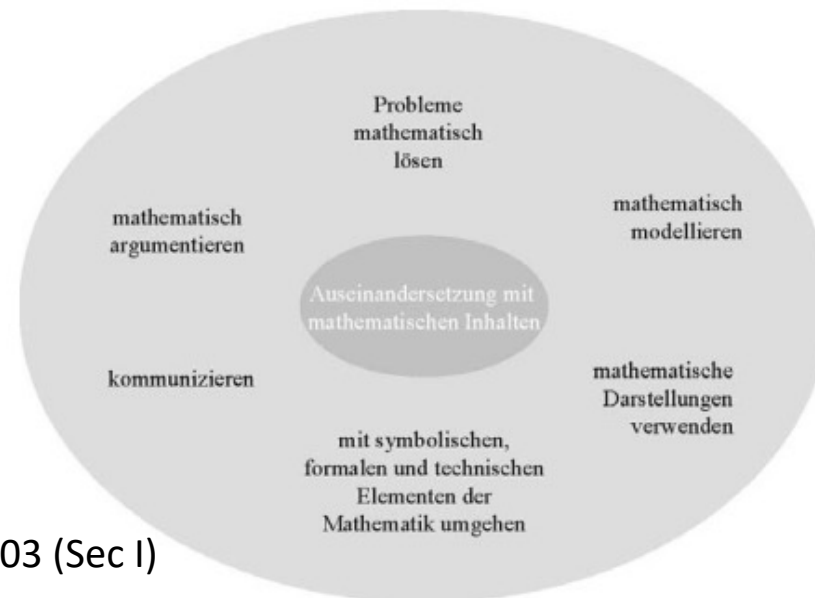
The story: Goals resulting from needs analysis

Coherence over the years – Mathematical Processes

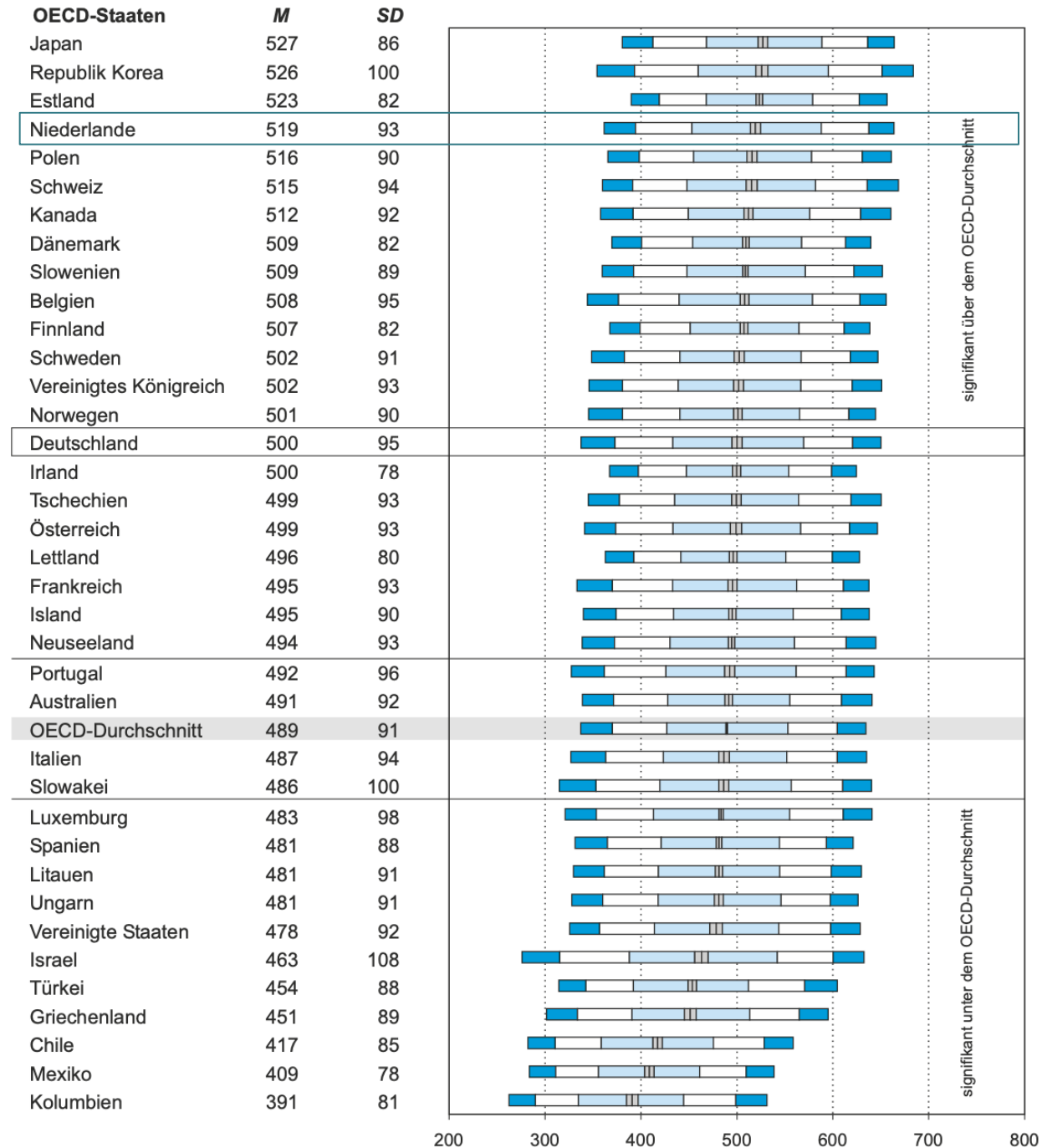
2022

Primary (1-4)	Secondary I (5 – 9/10)	Secondary II (10 - 12/13)
Mathematical arguing	Mathematical arguing	Mathematical arguing
Mathematical communicating	Mathematical communicating	Mathematical communicating
Solving problems mathematically	Solving problems mathematically	Solving problems mathematically
Mathematical modelling	Mathematisch modelling	Mathematical modelling
Mathematical representing	Mathematical representing	Using Mathematical representations
Working with mathematical objects and tools	Dealing with mathematical objects and tools	Dealing with symbolic, formal and technical elements of Mathematics
Working with media mathematically		

2003 (Sec I)



NEW Challenge



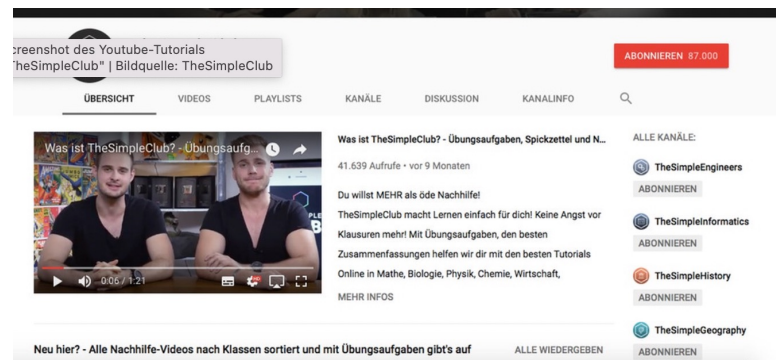
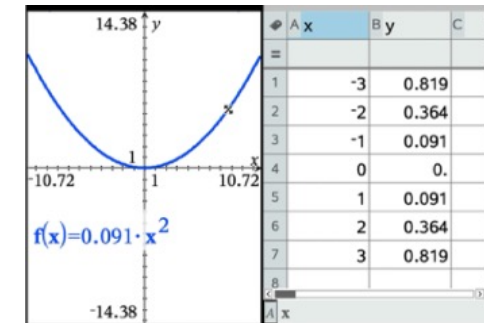
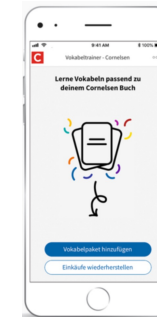
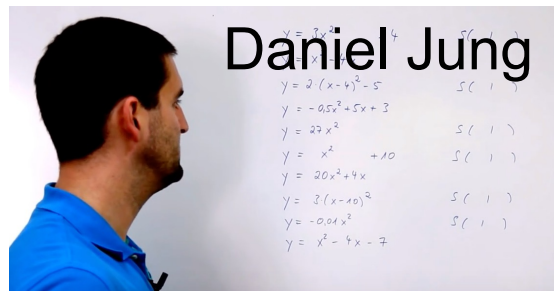
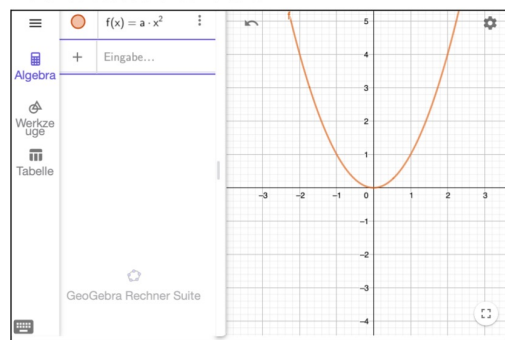
NEW Challenge: Education in the Digital World

Use of digital media remains far behind expectations

(Lorenz et al. 2017; Kuntze & Dreher 2013; Weigand 2014; Heid et al. 2013)

“quantitative and qualitative gap”

(Bretscher 2014, p. 43; Weigand, 2014; Heid, et al. 2013)



To integrate digital media in a meaningful way is a great challenge

NEW Challenge: Education in the Digital World

Use of digital media remains far behind expectations

(Lorenz et al. 2017; Kuntze & Dreher 2013; Weigand 2014; Heid et al. 2013)

Computer- und informations-
bezogene Kompetenzen
von Schülerinnen und
Schülern im zweiten
internationalen Vergleich und
Kompetenzen im Bereich
Computational Thinking

WAXMANN

ICILS 2018

#Deutschland

Birgit Eickelmann
Wilfried Bos
Julia Gerick
Frank Goldhammer
Heike Schaumburg
Knut Schwippert
Martin Senkbeil
Jan Vahrenhold
(Hrsg.)

Digital media are used with below-average frequency in the subjects in Germany in an international comparison.

The lowest percentage for Germany is found in mathematics (31.2%): Two thirds of 8th graders say they never work with media in Math. (Denmark: 96.9%)

Only 35% of teachers agree with the statement that the use of digital media can support pupils' learning processes.

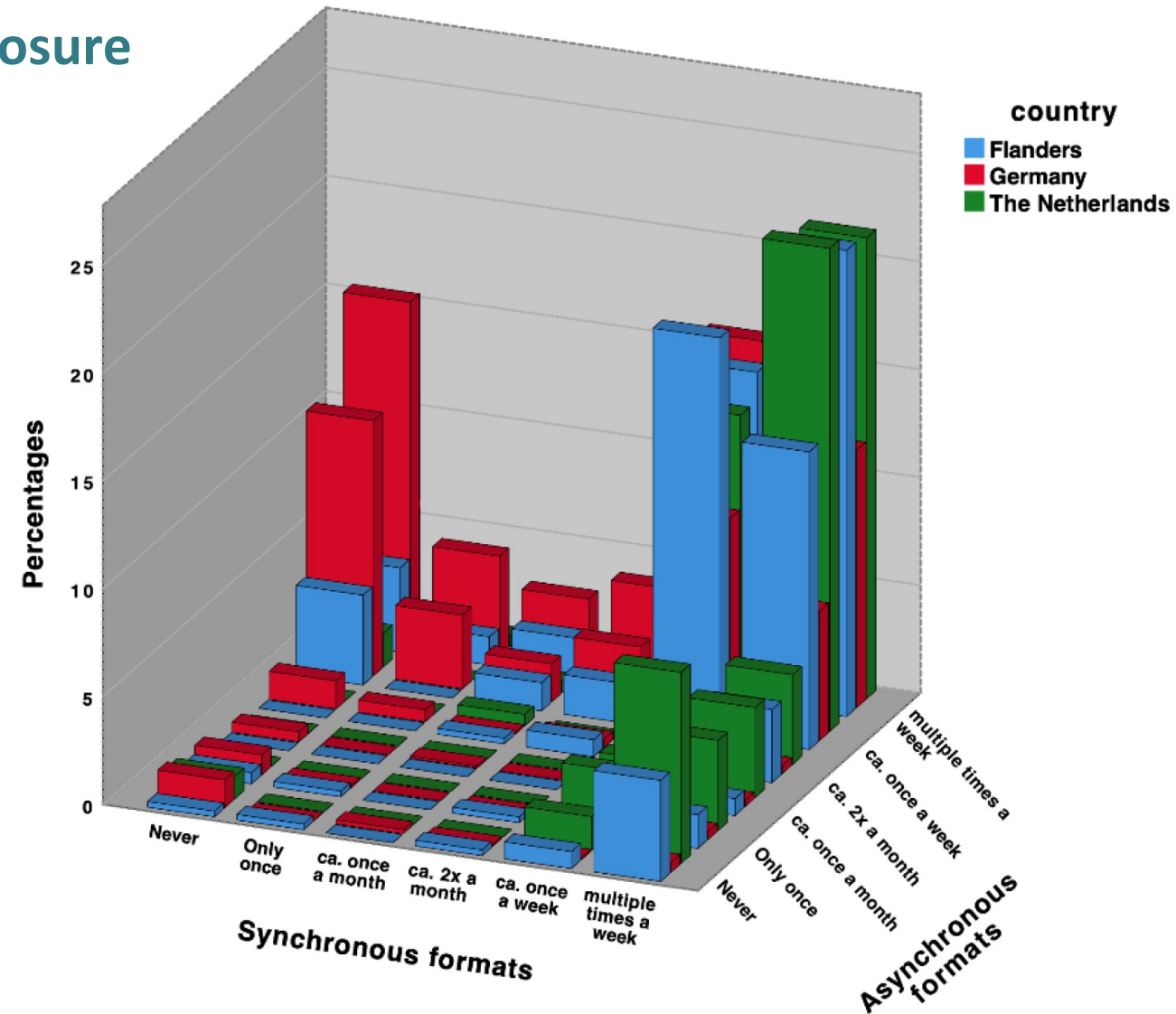
To integrate digital media in a meaningful way is a great challenge

New Challenge: Education in the Digital World

Learning mathematics in times of school closure

Math@distance

Study of the universities
Duisburg-Essen,
Utrecht &
Antwerpen with
1706 Math-Teachers



New Challenge: Core Results of Math@Distance

April 2020

April 2021



Significant development in the use of digital media (esp. For web conferencing) with increasing confidence in the use of digital media.



Maths-specific learning environments & diagnostic tools did not play an important role during the lockdown, although they were used before.



Decrease in classroom discussions as well as partner and group work phases.



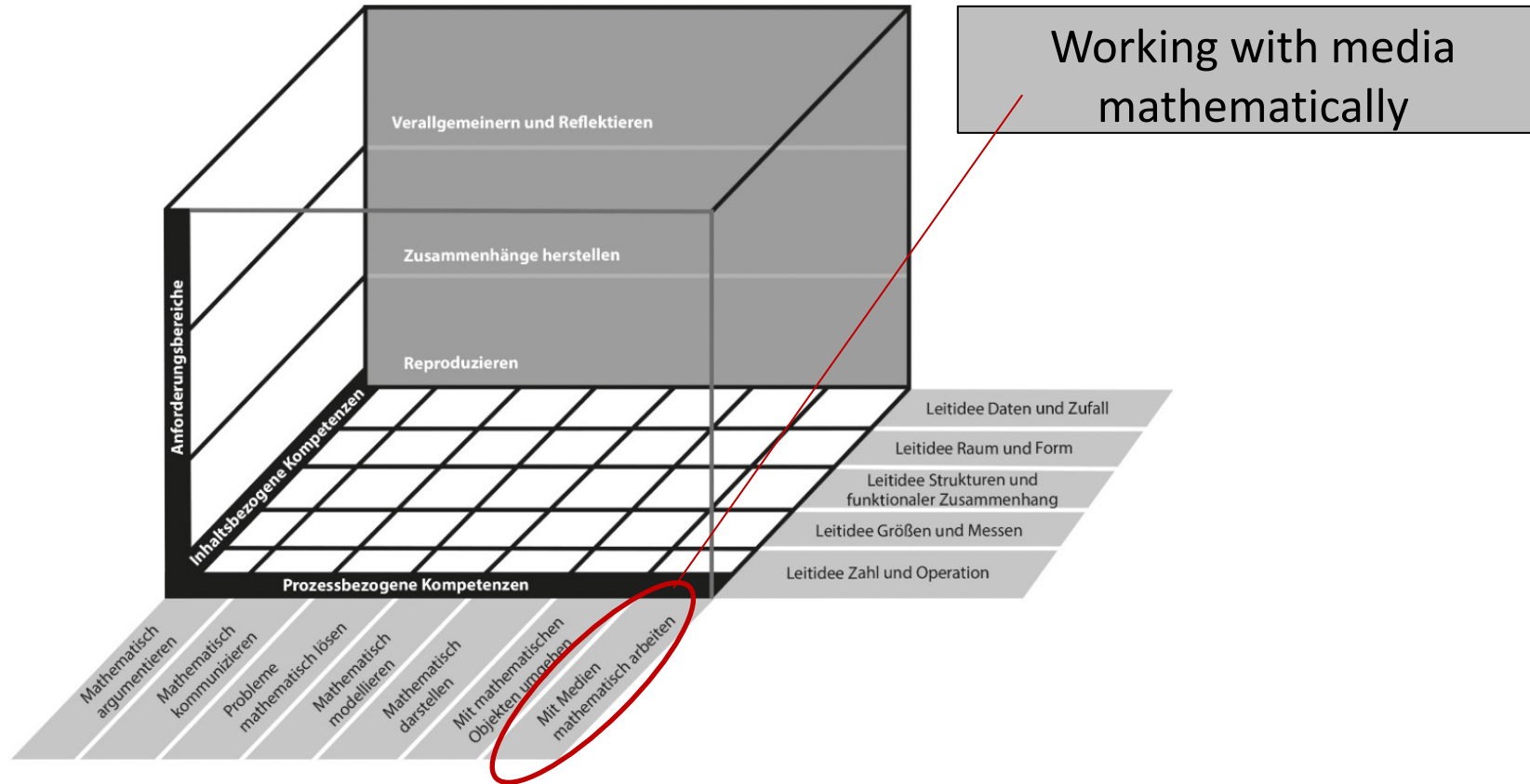
Focus on practice and numeracy, less on conceptual understanding (especially in Germany)



Importance of infrastructure and support



New Challenge: Education in the Digital World



Supporting mathematical competencies digitally
(Learning WITH media)
 &
 Supporting digital competencies mathematically
(Learning ABOUT media)

Abbildung 1: Kompetenzmodell der Bildungsstandards im Fach Mathematik für den Ersten Schulabschluss und den Mittleren Schulabschluss

Accompanying Pool of tasks on IQB-Webpage



<https://www.iqb.hu-berlin.de/bista/WeiterentwicklungBiSta/Lernaufgaben/MatheSekI>

New Challenge: Education in the Digital World

Working with media
mathematically

The range of students' competences covers:

- the **use** of general media (analogue & digital)
- the **critical perception of information** from the digital world from a mathematical point of view,
- the **use of digital mathematics tools** and learning environments, and
- the **creation and design** of general media (e.g. videos presentations)
- the **development and reflection of algorithms** with the help of digital media.

Supporting mathematical
competencies digitally
(Learning WITH media)
&
Supporting digital
competencies mathematically
(Learning ABOUT media)

New Challenge: Using the whole range of Digital Media

Learning **WITH** Media:

Media are used in a reflective way to **support cognitive activities in learning or to support learning or enable new ones**

Learnin **ABOUT** Media:

Pupils **get to know media, select them according to the goal, adapt them** if necessary and deal with them consciously and critically.

Digital Media

Digital Learning environments

(e.g. Videos, Interactive environments)

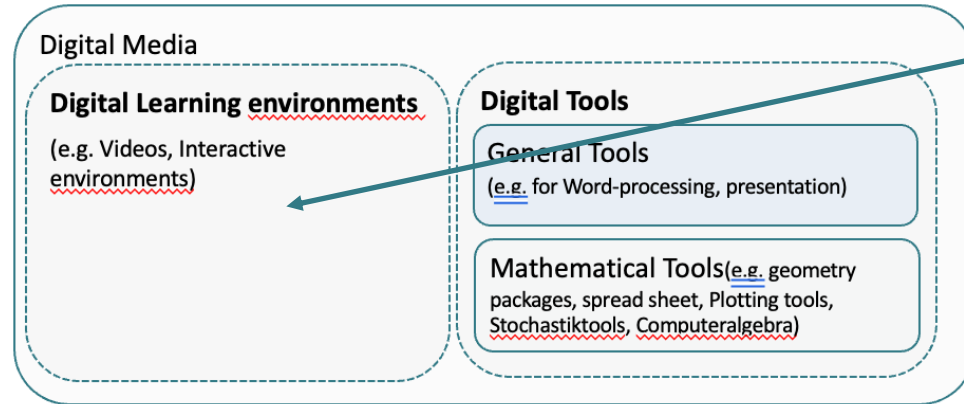
Digital Tools

General Tools

(e.g. for Word-processing, presentation)

Mathematical Tools(e.g. geometry packages, spread sheet, Plotting tools, Stochastiktools, Computeralgebra)

Illustrative Examples



Digital Learning environments
(e.g. Videos, Interactive environments)

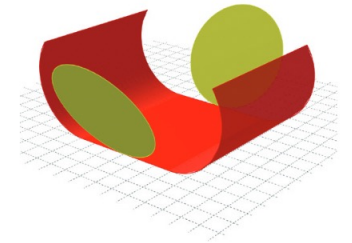
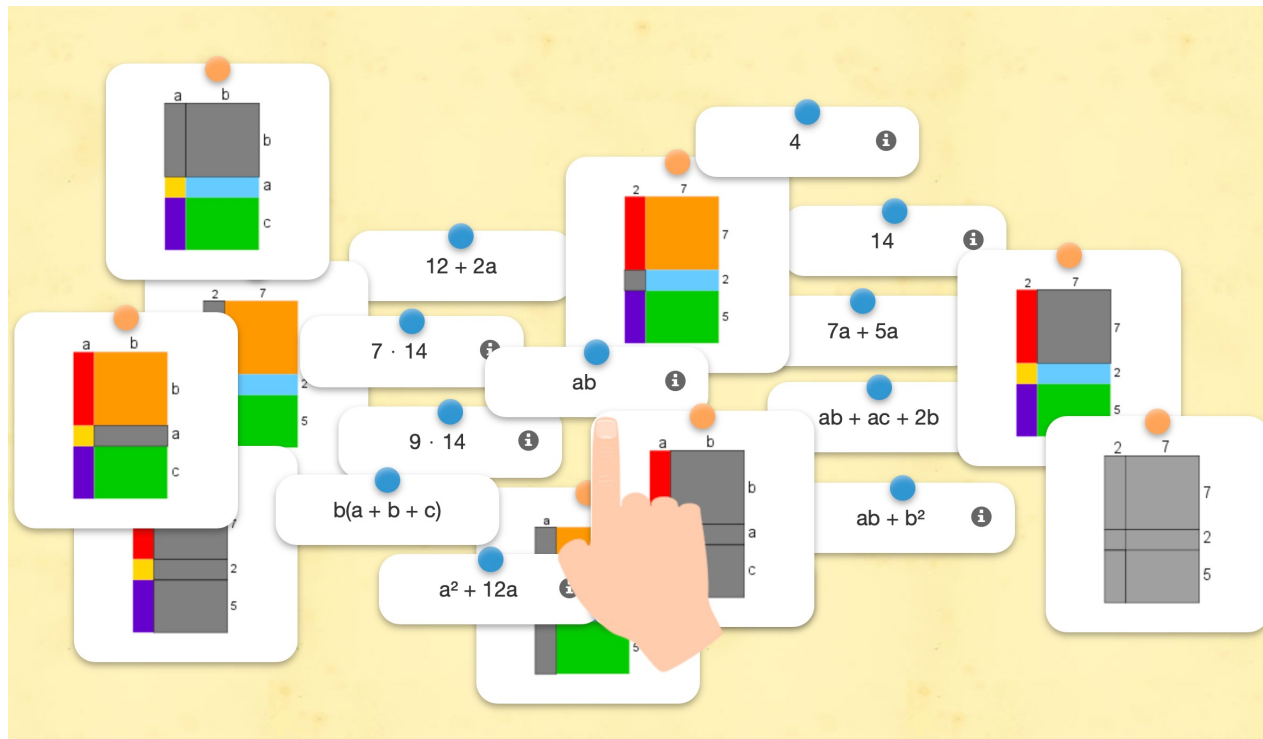
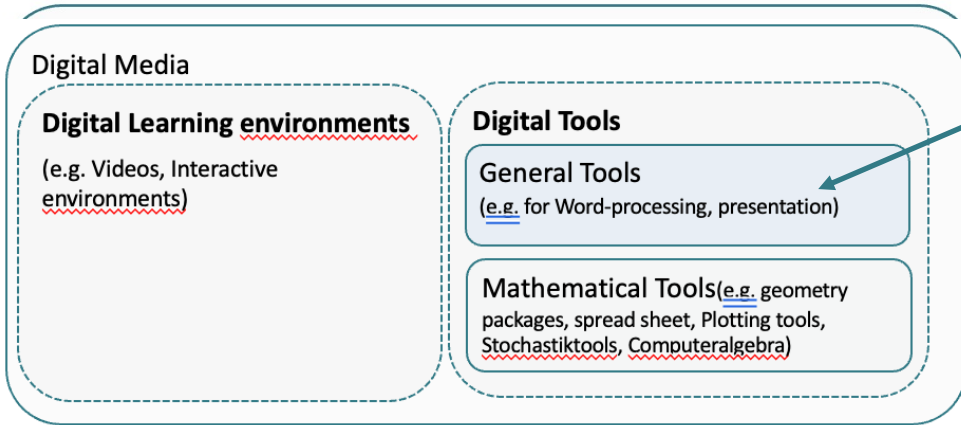


Abb. 6: (a) Idealisiertes Modell mit AR und (b) Abwicklung des Umwickelpapiers (erstellt mit GeoGebra 3D Rechner, Datei «Abrollen des Netzes eines Zylinders» von Birgit Lachner, <https://www.geogebra.org/search/Zjr38uff>).

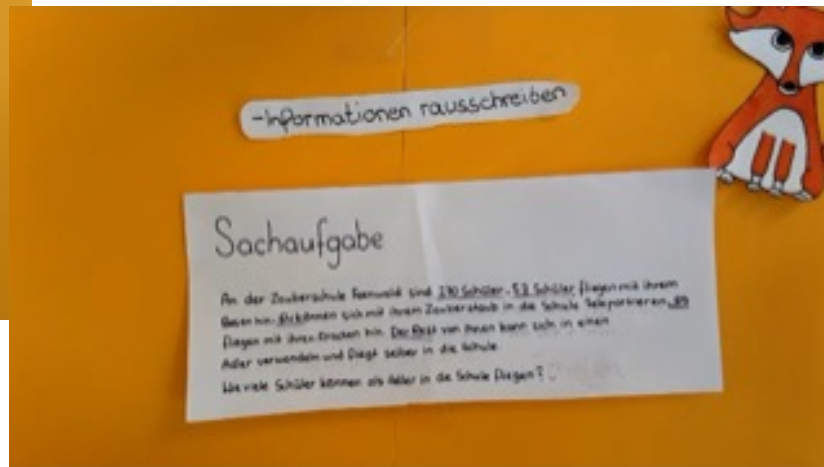


Illustrative Examples



General Tools
(e.g. for Word-processing, presentation)

Students as recipient & producers of videos



MINT-Congress 2018 at Uni Duisburg-Essen:



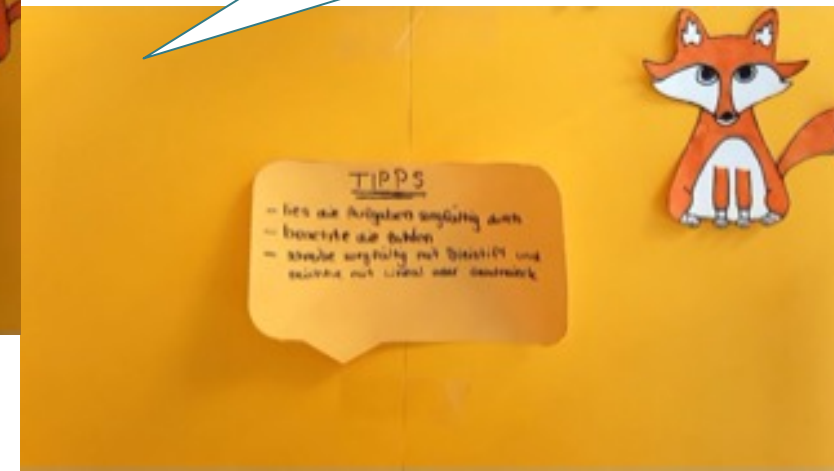
Bruchschule, Ruhr-Gymnasium, Vormholzer Grundschule - Witten



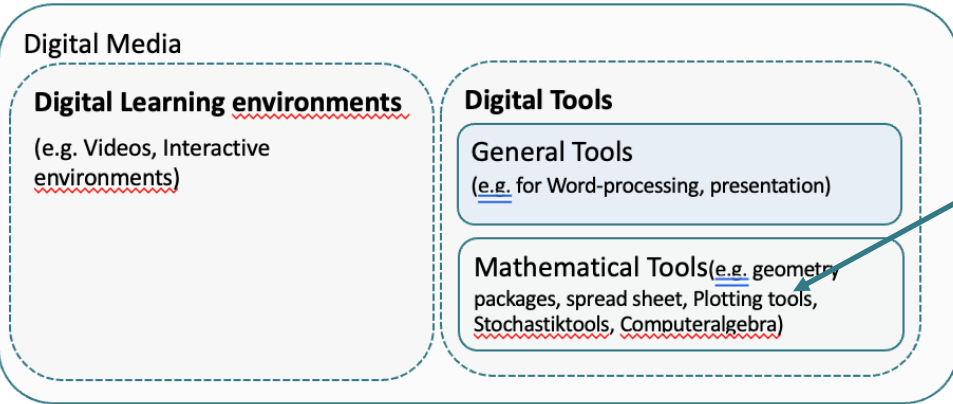
MINT-ovate



„...they should have an idea in mind to understand it....“



Illustrative Examples



Mathematical Tools (e.g. geometry packages, spread sheet, Plotting tools, Stochastiktools, Computeralgebra)

$$x \text{ km} \cdot 0,04 \text{ l/km} \cdot 1,50 \text{ €}$$



Was kostet das Autofahren im Vergleich zum Bahnfahren?

Die Berechnung ist fast immer gleich. Meistens verändert sich nur die Kilometerzahl. Der Term dazu sieht so aus:
 $x \cdot 0,04 \cdot 1,50 = y$

Tabellenkalkulationsblatt zur Aufgabe Erkunden 4: Was kostet das Auto?					
Wertverlust in 4 Jahren (in Euro)	Benzinpreis (in Euro pro Liter)	Kilometerzahl in 4 Jahren (in Kilometer)	Benzinkosten in 4 Jahren (in Euro)	Jährliche Reparatur / Steuer / Versicherung (in Euro)	Gesamtkosten in 4 Jahren (in Euro)
3000	2,05	40000	3280	900	9880



Story & Challenge: Program coherence to guide School Improvement



**KULTUSMINISTER
KONFERENZ**

Often described challenge in TPD structures

Programs often fragmented, i.e.

- each TPD sends different messages
- connections hard to recognize
- teachers feel overload

Too much inert knowledge

- TPD course often do not prepare teaching practices
- Pedagogical content knowledge and teaching practices too unconnected

Facilitators miss coherent orientations

- missing coherent vision of quality math classrooms
- single TPD content not part of a larger whole
- missing connections

*Educational Evaluation and Policy Analysis
Winter 2001, Vol. 23, No. 4, pp. 297–321*

Instructional Program Coherence: What It Is and Why It Should Guide School Improvement Policy

Fred M. Newmann

University of Wisconsin, Madison

BetsyAnn Smith

Michigan State University

Elaine Allensworth

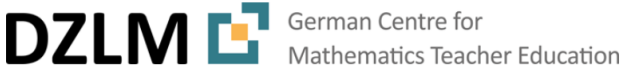
Consortium on Chicago School Research

Anthony S. Bryk

University of Chicago


We present the concept of instructional program coherence and explain why school improvement frameworks that incorporate instructional program coherence are more likely to advance student achievement than multiple, unrelated efforts. We present evidence that Chicago elementary schools with stronger instructional program coherence make higher gains in student achievement. We also share observations on how, in specific schools, principals and external partners directed key school resources toward the development of instructional program coherence. In closing, we discuss factors within the educational system that discourage instructional program coherence and suggest ways that school leaders, school improvement partners, and policymakers can support greater instructional program coherence.


Story & Challenge: Program coherence to guide School Improvement





Coherent vision for high-quality mathematics teaching


QuaMath  Unterrichts- und Fortbildungs-Qualität
in Mathematik entwickeln

 **Cognitive demand:**
Initiate active learning processes

 **Focus on understanding:** of concepts,
strategies and procedures

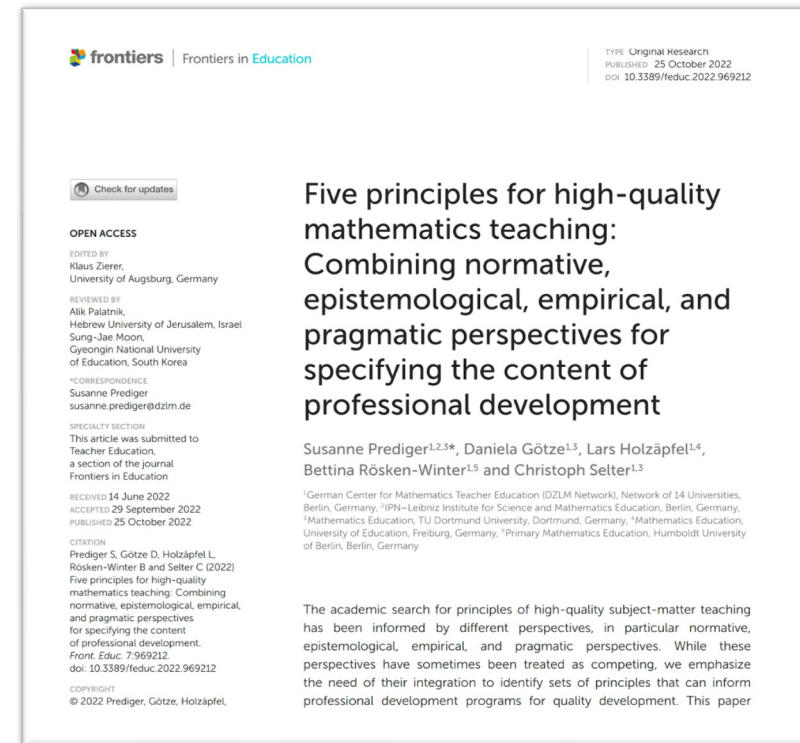
 **Longitudinal coherence:**
Prepare for sustainable learning

 **Student focus & adaptivity:**
Work with student perspectives

 **Enhanced communication:**
Talk about mathematics

Principles selected and articulated as coherent quality dimensions by

- intensive literature research
- tested in use (e.g. as coherent core for novice facilitators' practices)
- negotiated with the DZLM network and the state coordinators (teachers & stakeholders)



frontiers | Frontiers in Education

TYPE Original Research
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[Check for updates](#)

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and pragmatic perspectives
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of professional development.
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Five principles for high-quality mathematics teaching: Combining normative, epistemological, empirical, and pragmatic perspectives for specifying the content of professional development


Susanne Prediger^{1,2,3*}, Daniela Götze^{1,3}, Lars Holzäpfel^{1,4}, Bettina Rösken-Winter^{1,5} and Christoph Selter^{1,3}


¹German Center for Mathematics Teacher Education (DZLM Network), Network of 14 Universities, Berlin, Germany, ²IPN—Leibniz Institute for Science and Mathematics Education, Berlin, Germany, ³Mathematics Education, TU Dortmund University, Dortmund, Germany, ⁴Mathematics Education, University of Education, Freiburg, Germany, ⁵Primary Mathematics Education, Humboldt University of Berlin, Berlin, Germany


The academic search for principles of high-quality subject-matter teaching has been informed by different perspectives, in particular normative, epistemological, empirical, and pragmatic perspectives. While these perspectives have sometimes been treated as competing, we emphasize the need of their integration to identify sets of principles that can inform professional development programs for quality development. This paper


Coherent vision for high-quality mathematics teaching


QuaMath  Unterrichts- und Fortbildungs-Qualität
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**Digitalisation offers an
opportunity for all
principles!**

Illustrative Example: Space and Shape



Cognitive demand:
Initiate active learning processes



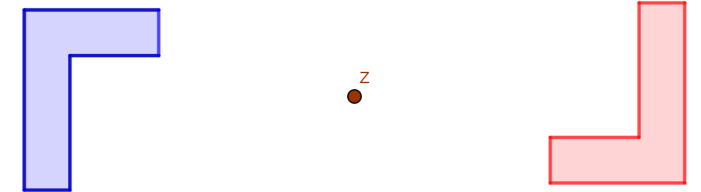
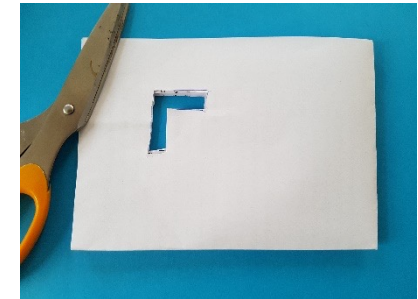
Student focus & adaptivity:
Work with student perspectives



Focus on understanding: of concepts, strategies and procedures



Enhanced communication:
Talk about mathematics



Die Schülerinnen und Schüler...

Raum und Form:

- entwickeln Vorstellungen im zwei- und dreidimensionalen Raum und operieren (insbesondere verschieben, drehen, spiegeln) gedanklich mit den darin enthaltenen Objekten (Punkten, Strecken, Flächen und Körpern).

mathematisch darstellen:

- nutzen und erzeugen vertraute und geübte Darstellungen von mathematischen Objekten und Situationen (AFB I)

mit Medien mathematisch arbeiten:

- nutzen analoge und digitale Mathematikwerkzeuge (z. B. Geometriesoftware, Tabellenkalkulation, Computeralgebrasystem, Stochastiktool1) zum Problemlösen, Entdecken, Modellieren, Daten verarbeiten, Kontrollieren und Darstellungswechseln etc., (AFB II)

inhaltsbezogene Kompetenz

(Nicht angesprochene Bereiche der Teilkompetenz werden ausgegraut.)

prozessbezogene Kompetenzen (AFB)

Investigate whether a single axis mirroring can be performed instead of the point mirroring with the same result.

Illustrative Example: Structures and Functional dependency



Cognitive demand:
Initiate active learning processes



Focus on understanding: of concepts, strategies and procedures



Student focus & adaptivity:
Work with student perspectives



Enhanced communication:
Talk about mathematics

„.....implement known mathematical procedures as algorithms using digital tools (e.g. spreadsheets)“

*Any two fractions are to be multiplied together.
Each fraction is to be displayed as a fraction*

- (i) Create the machine.....*
- (ii) Check that it works correctly. ...*
- (iii) Save your file.....*

	A	B	C	D	E
10	Multiplikation von Brüchen				
11					
12	3	*	9	=	27
13	8		5		40
14					
15					

„Fraction-Calculating-Machine“

Illustrative Example: Structures and Functional dependency

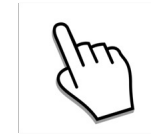


Cognitive demand:
Initiate active learning processes

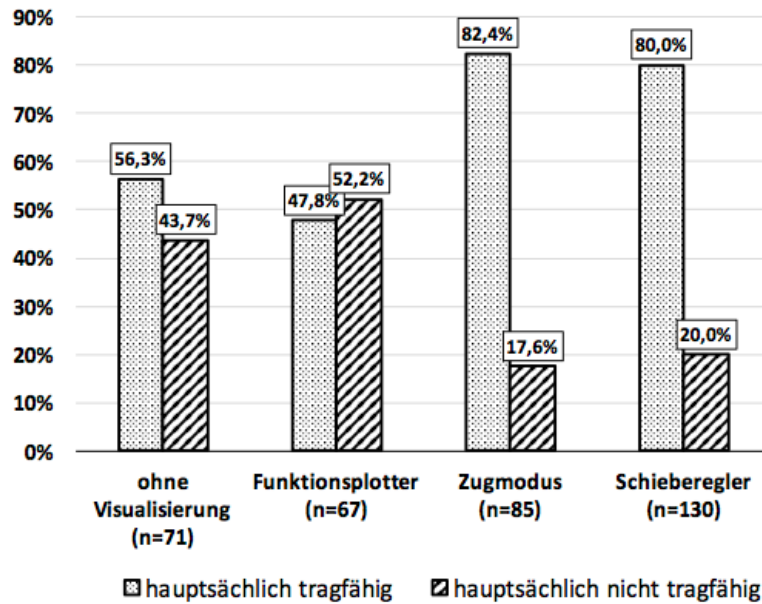


Focus on understanding: of concepts, strategies and procedures

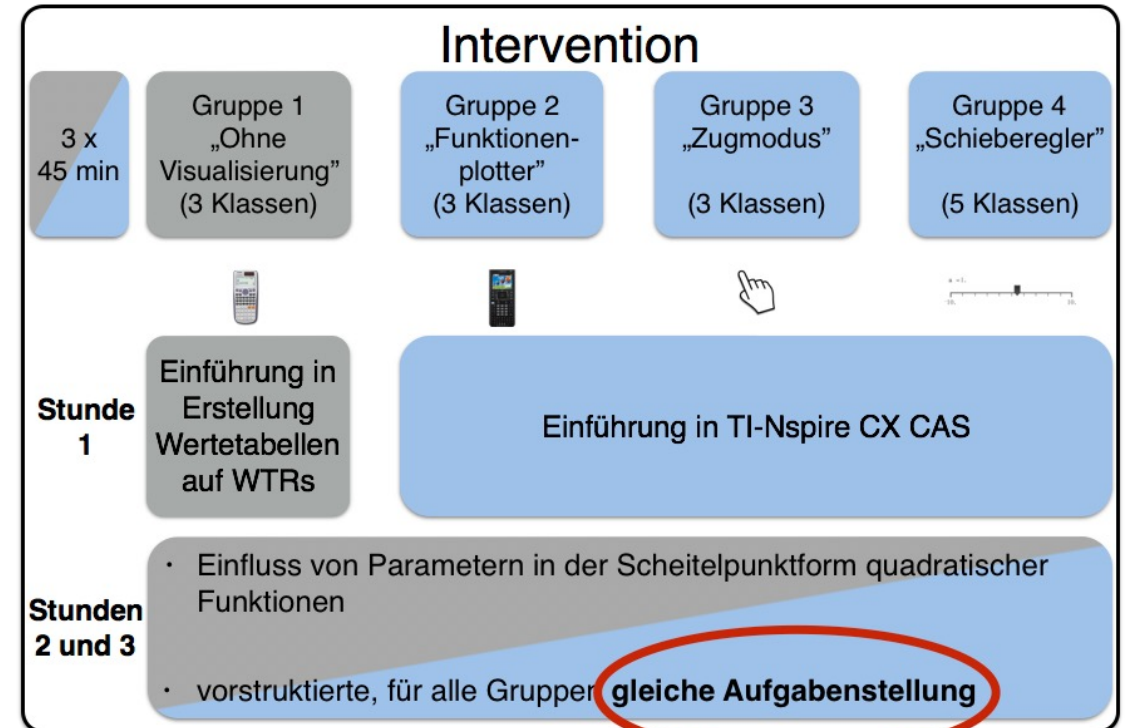
Investigating the meaning of the parameters a, b and c in $f(x) = a(x - b)^2 + c$



Tragfähigkeit der Merkblätter



The groups with dragmode and sliders note much more meaningful answers on their summary sheets



Illustrative Example: Structures and Functional dependency



Cognitive demand:
Initiate active learning processes



Focus on understanding: of concepts, strategies and procedures

Investigating the meaning of the parameters a,b and c
in $f(x) = a(x - b)^2 + c$

Parameter	Category	Without visualization	Function Plotter	Drag mode	Sliders
a	Vertical Shrink	Yellow	Red	Green	Green
	Vertical Stretch	Yellow	Red	Green	Yellow
	Reflection	Yellow	Red	Yellow	Green
	Case $a = 0$	Red	Red	Yellow	Yellow
b	Right	Red	Red	Yellow	Green
	Left	Red	Red	Yellow	Green
	Horizontal	Red	Yellow	Green	Green
c	Upwards	Red	Red	Yellow	Yellow
	Downwards	Yellow	Red	Yellow	Yellow
	Vertical	Yellow	Yellow	Green	Green

The potentials and burdens are much more manifold than expected
Sliders play the role of an „external“ variable

Illustrative Example: Structures and Functional dependency



Focus on understanding: of concepts, strategies and procedures

... know about basic ideas
(Grundvorstellungen)
and link relevant representations

...knowing about the meaning and sense

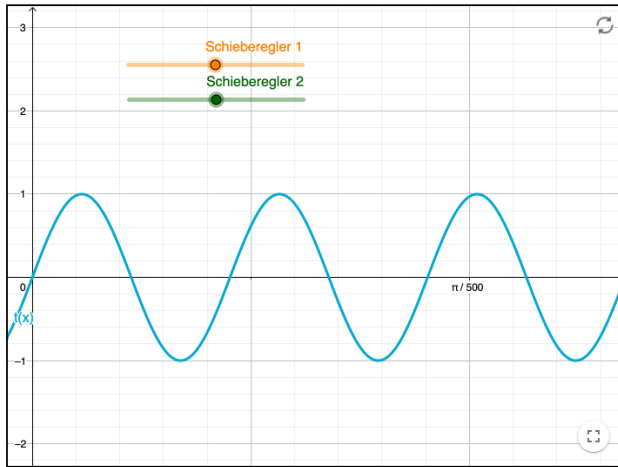
Aufgabentitel	Töne mit Sinusfunktionen modellieren und untersuchen		
Ziele der Aufgabe	Die Schülerinnen und Schüler entdecken die Bedeutung der Parameter der Sinusfunktion im musikalischen Kontext von Tönen, anhand der App Mathe-Synthesizer.		
Bildungsstufe Klassenstufe	<input type="checkbox"/> ESA <input checked="" type="checkbox"/> MSA <input type="checkbox"/> Beide 10	Bearbeitungszeit gesamt in Minuten	45-60 Minuten
Leitidee 1	Strukturen und funktionaler Zusammenhang	Leitidee 2	Bitte wählen
Einsatz von (digitalen) Medien	App: „Mathe- Synthesizer“	Unterrichtsphase	1,2: Entdecken/Einstieg 2: Systematisieren

Students discover the meaning of parameters at sine function in the context of sound & music

Illustrative Example: Structures and Functional dependency



Focus on understanding: of concepts, strategies and procedures



Einfluss der Schieberegler | Beschreibung

Aa π Gib hier deine Antwort ein...

Beschriftung: Schieberegler 1

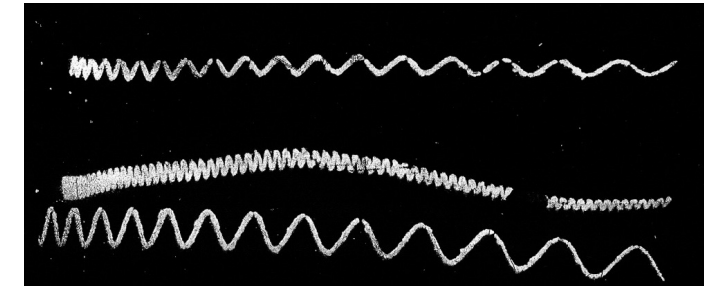
Aa π Gib hier deine Antwort ein...

Beschriftung: Schieberegler 2

Aa π Gib hier deine Antwort ein...

← Vorherig
Den Sinus hören? - Transformationen der Sinus-Funkt...

Weiter →
Ein kleines Tonlabor



$f(t) = a \cdot \sin(b \cdot t) =$

Graph

Audio

MacBook Pro-Lautspr...

Nicolas Regel, TU Dresden

Illustrative Example: Data and Chance



Cognitive demand:
Initiate active learning processes



Student focus & adaptivity:
Work with student perspectives



Focus on understanding: of concepts, strategies and procedures



Enhanced communication:
Talk about mathematics

Conditional probability
as new topic in Secondary I

inhaltsbezogene Kompetenz

(Nicht angesprochene Bereiche der Teilkompetenz werden ausgegraut.)

Die Schülerinnen und Schüler...

Daten und Zufall:

Werten grafische Darstellungen und Tabellen von statistischen Erhebungen aus, auch mit Hilfe von Tabellenkalkulation oder Stochastiktools

Nutzen Visualisierungen, um bei einfachen alltagsnahen Modellierungen bedingte Wahrscheinlichkeiten zu erkennen, ohne und mit Hilfe digitaler Medien (MSA)

ODER:

Zahl und Operation:

nutzen sinntragende Vorstellungen von rationalen Zahlen, insbesondere von natürlichen, ganzen und gebrochenen Zahlen entsprechend der Verwendungsnotwendigkeit,

erläutern an Beispielen die verschiedenen Vorstellungen zum Bruchbegriff (insbesondere Teile eines oder mehrerer Ganzer, relative Anteile),

Mathematisch kommunizieren:

gehen fachbezogen auf Äußerungen von anderen zu mathematischen Inhalten ein (z. B. konstruktiver Umgang mit Fehlern, Weiterführen mathematischer Ideen)

Mathematisch argumentieren:

bewerten Ergebnisse und Aussagen auch bzgl. ihres Anwendungskontextes

Mathematisch darstellen:

wechseln sachgerecht zwischen mathematischen Darstellungen und erklären, wie sie vernetzt sind

prozessbezogene Kompetenzen

Students use visualisations to identify conditional probabilities in simple everyday modelling....

Illustrative Example: Data and Chance



Cognitive demand:
Initiate active learning processes



Student focus & adaptivity:
Work with student perspectives



Focus on understanding: of concepts,
strategies and procedures



Enhanced communication:
Talk about mathematics

Conditional probability
as new topic in Secondary I



A New Visualization for Probabilistic Situations Containing Two Binary Events: The Frequency Net

Karin Binder*, Stefan Krauss and Patrick Wiesner

Mathematics Education, Faculty of Mathematics, University of Regensburg, Regensburg, Germany

In teaching statistics in secondary schools and at university, two visualizations are primarily used when situations with two dichotomous characteristics are represented: 2×2 tables and tree diagrams. Both visualizations can be depicted either with probabilities or with frequencies. Visualizations with frequencies have been shown to help students significantly more in Bayesian reasoning problems than probability visualizations do. Because tree diagrams or double-trees (which are largely unknown in school) are node-branch structures, these two visualizations (in contrast to the 2×2 table) can even simultaneously display probabilities on branches and frequencies inside the nodes. This is a teaching advantage as it allows the frequency concept to be used to better understand probabilities. However, 2×2 tables and (double-)trees have a decisive disadvantage: While *joint probabilities* [e.g., $P(A \cap B)$] are represented in 2×2 tables but no *conditional probabilities* [e.g., $P(A|B)$], it is exactly the other way around with (double-)trees. Therefore, a visualization that is equally suitable for the representation of joint probabilities and conditional probabilities is desirable. In this article, we present a new visualization—the *frequency net*—in which all absolute frequencies and all types of probabilities can be depicted. In addition to a detailed theoretical analysis of the frequency net, we report the results of a study with 249 university students that shows that “net diagrams” can improve reasoning without previous instruction to a similar extent as 2×2 tables and double-trees. Regarding

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Students use visualisations to identify conditional probabilities in simple everyday modelling....

Illustrative Example: Data and Chance

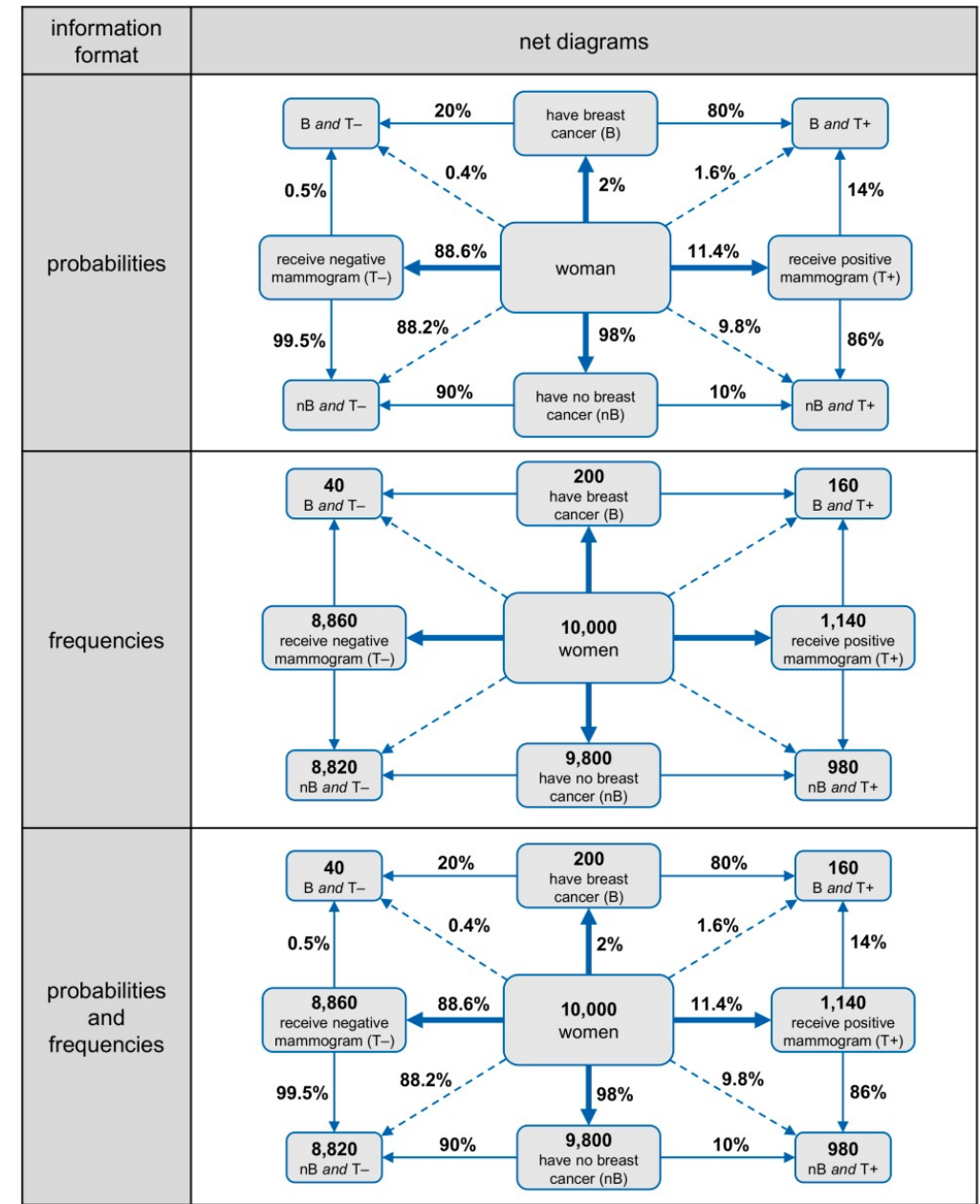
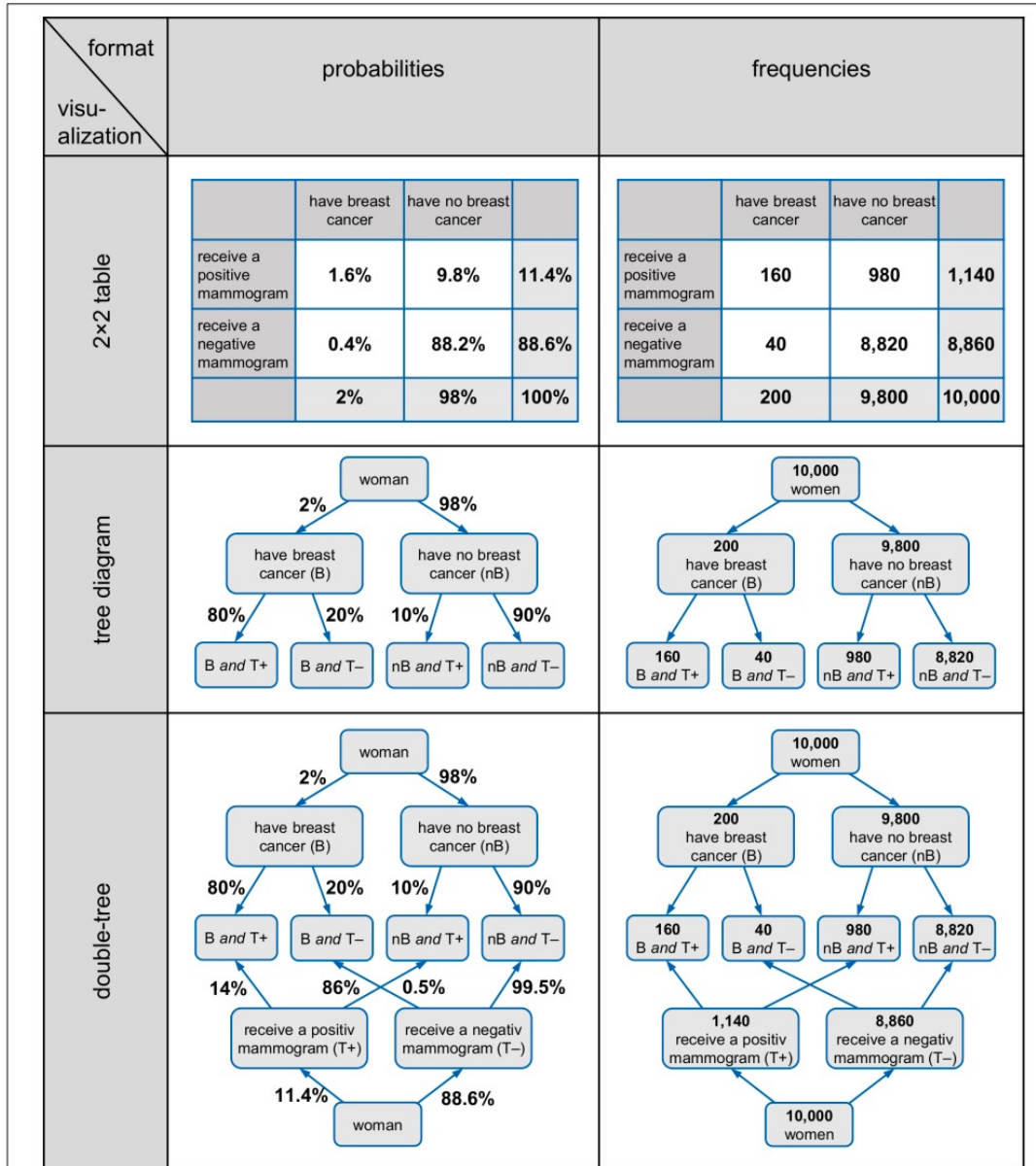
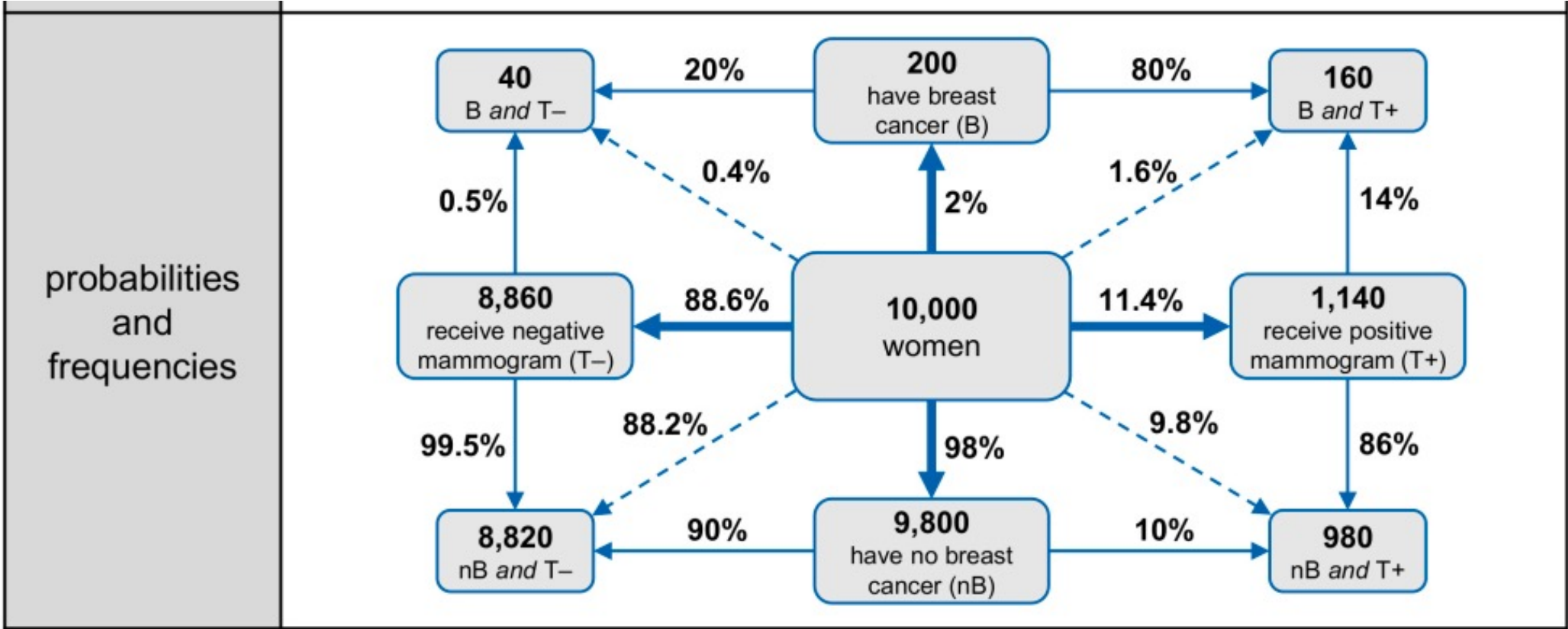


FIGURE 1 | 2 × 2 tables, tree diagrams, and double-trees (left in probabilities, right in frequencies) for the mammography problem.

Illustrative Example: Data and Chance

Frequency net



Illustrative Example: Data and Chance

?! Cognitive demand:
Initiate active learning processes

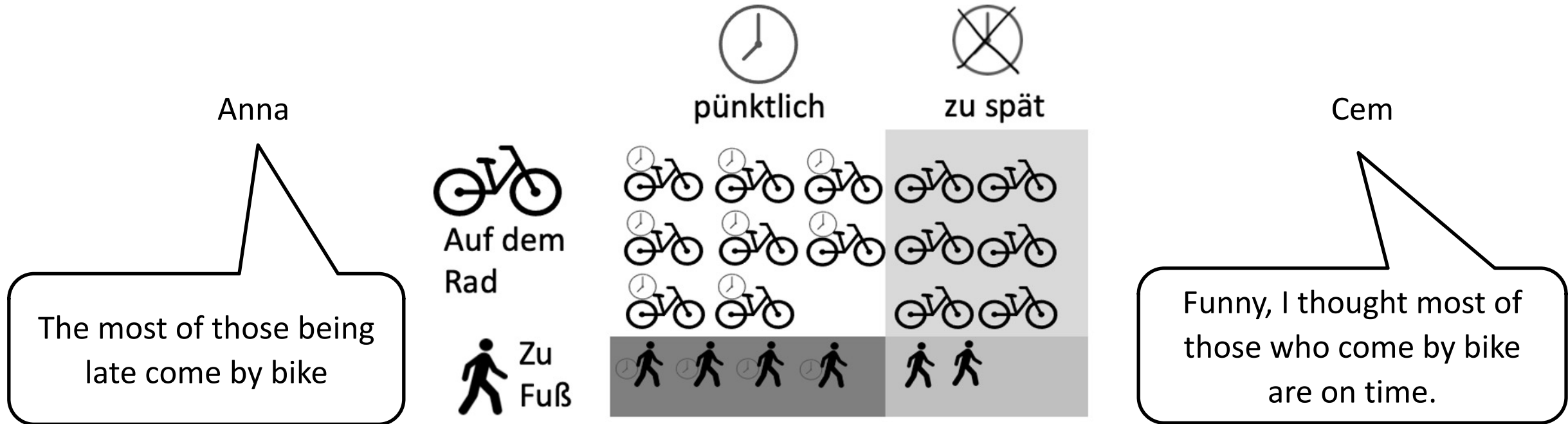
Student focus & adaptivity:
Work with student perspectives

**Conditional probability
as new topic in Secondary I**

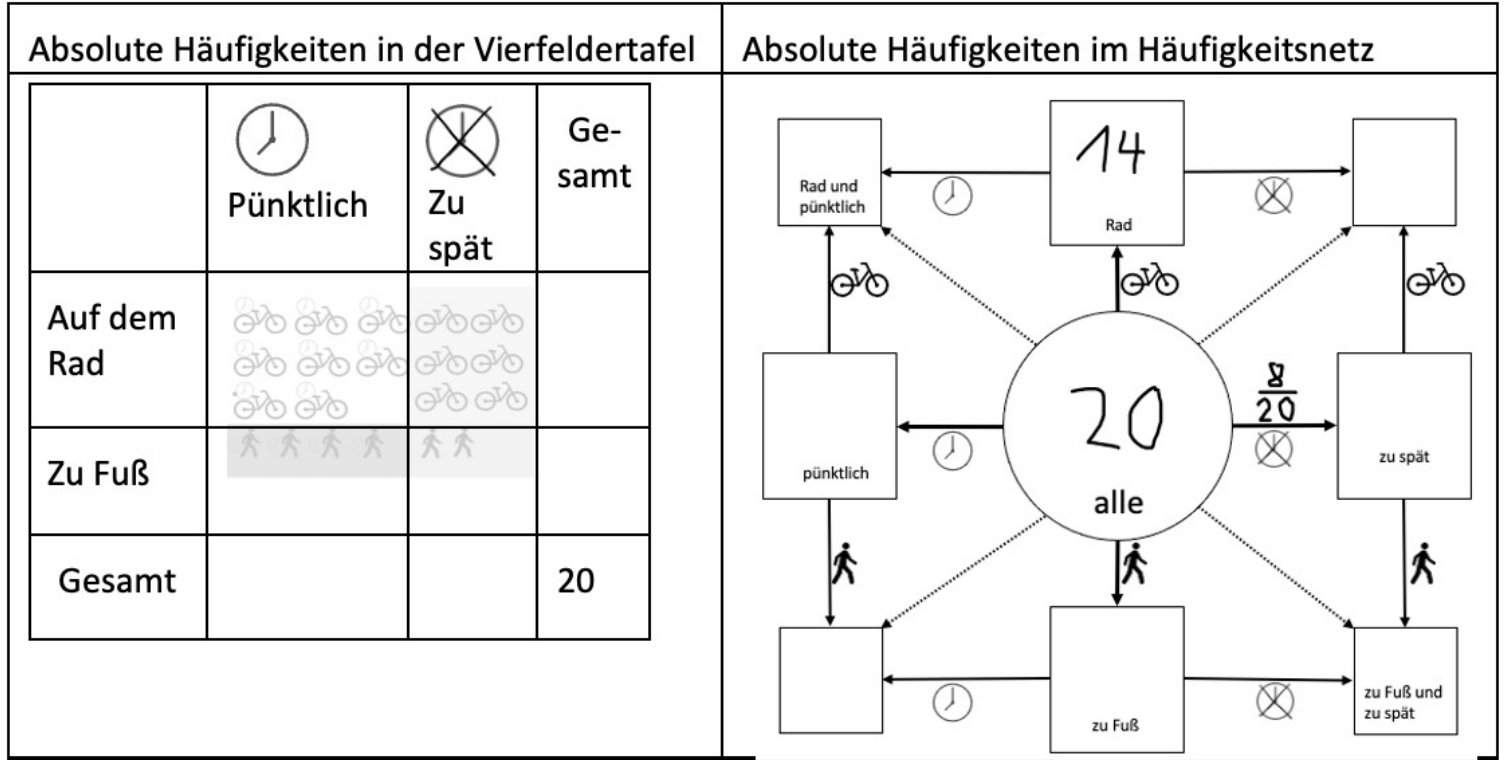
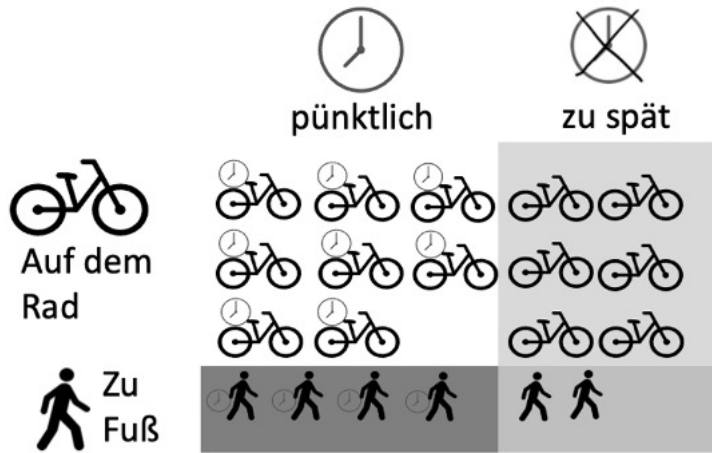
Focus on understanding: of concepts,
strategies and procedures

Enhanced communication:
Talk about mathematics

In Cem and Anna's class, pupils are often late. There is a heated discussion about who is actually late. The class has conducted a survey on this. There are 20 children in the class.

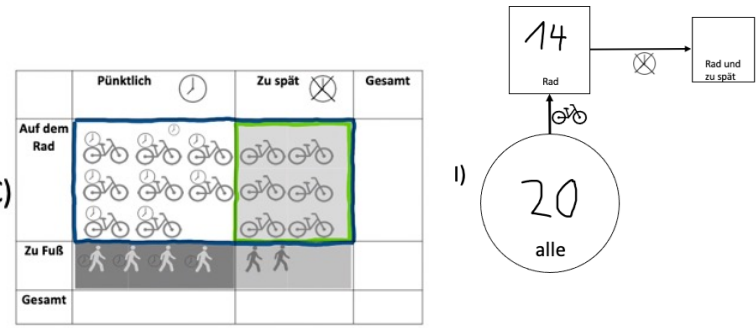


Illustrative Example: Data and Chance



$$\frac{6}{14}$$

The part of cyclists who arrive late



Coherence over the years

Mathematical Tools (e.g. geometry packages, spread sheet, Plotting tools, Stochastiktools, Computeralgebra)

Finanzierungsplan für Möbel		1. Jahr	2. Jahr	3. Jahr
Jährliche Rate	Zinssatz pro Jahr	Schulden zu Jahresbeginn	Zinsen	Rate
1.800,00 €	4,95%	10.000,00 €	495,00 €	1.800,00 €
		Restschuld am Jahresende		
		8.695,00 €	7.325,40 €	

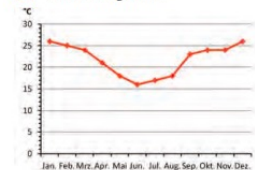
Tabellenkalkulationsblatt zur Aufgabe Erkunden 4: Was kostet das Auto?

Wertverlust in 4 Jahren (in Euro)	Benzinpreis (in Euro pro Liter)	Kilometerzahl in 4 Jahren (in Kilometer)	Benzinkosten in 4 Jahren (in Euro)	Jährliche Reparatur / Steuer / Versicherung (in Euro)	Gesamtkosten in 4 Jahren (in Euro)
3000	2,05	40000	3280	900	9880

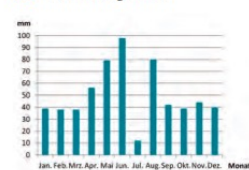


Multiplikation von Brüchen				
3	*	9	=	27
8		5		40

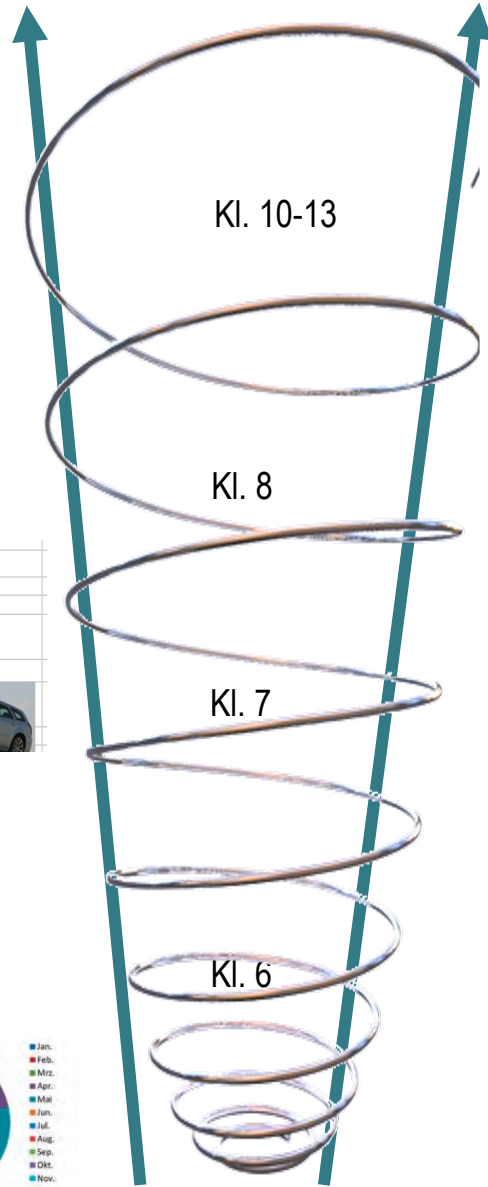
(1) Liniendiagramm



(2) Säulendiagramm



(3) Kreisdiagramm



Kl. 10-13

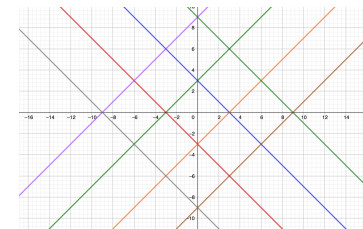
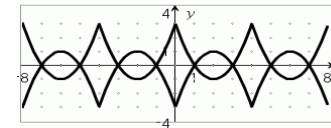
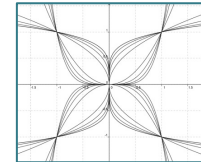
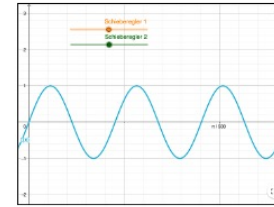
Kl. 8

Kl. 7

Kl. 6



Longitudinal coherence:
Prepare for sustainable learning



As well for

- Geometry/ DGS
- Stochastics/ ST

Digital diagnostics in Germany



Student focus & adaptivity:
Work with student perspectives

Diagnosis often just as percentages of correctly solved tasks

Teilnehmer		8		>
Asta Kraushaar	-	15		>
Berend Otto	95%	15		>
Claus Fritsch	55%	15		>
Denny Weitzel	-	15		>
Eugenia Trommler	0%	15		>
Evangelos Bohlander	100%	15		>
Hermann Josef Weiß	100%	15		>
Leonore Hamann	-	15		>
Lotte Steckel	95%	15		>
Margaret Ackermann	-	15		>
Marjan Thanel	-	15		>
Reinhold Ullrich	59%	15		>
Siegfried Stoll	-	15		>
Susan Ruppert	-	15		>
Verena Huhn	-	15		>

Ergebnisse von Claus Fritsch				
09.07.2018	55%	12/22 Pkt.		Bestes Ergebnis
09.07.2018	0%	0/22 Pkt.		

Encouragement to repeat the same tasks

Aufgabe 5

Berechne.

77 - 20 =

Lösungsschritte:

1. Subtrahieren ohne Übertrag

Solution frequencies of procedural tasks

Aufgabe 5  57%

Digital diagnostics in Germany

bettermarks⁷[®]
ERFOLGREICH MATHE LERNEN

ANTON

Mathegym
Mathe-Online-Training

Focus on procedures

Diagnosis often just right or wrong

Intensifies current problems

in Mathematics Teaching:

- Focus on procedures
- Superficial Diagnosis



Student focus & adaptivity:
Work with student perspectives

SMART

Specific **M**athematics **A**ssessments
that **R**eveal **T**hinking



**KULTUSMINISTER
KONFERENZ**

Focus on understanding

Diagnosis of levels of understanding,
misconceptions & hints for individual
support

Offers Hints to overcome current problems
in Mathematics Teaching:

- Understanding
- Deep Diagnosis
- Professionalisation of teachers

MINTUS – Beiträge zur
mathematisch-naturwissenschaftlichen Bildung

RESEARCH

Daniel Thurm · Laura A. Graewert

**Digitale
Mathematik-
Lernplattformen in
Deutschland**

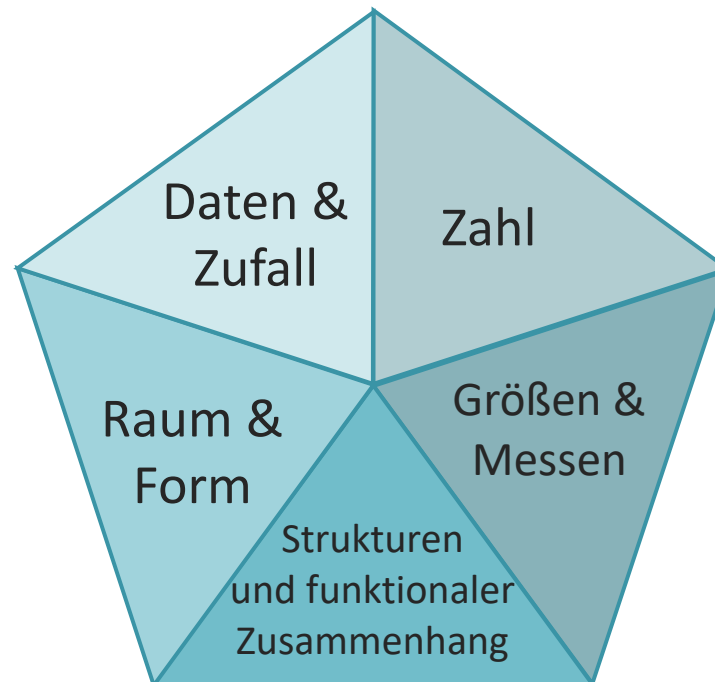
Springer Spektrum



SMART

SMART is a project of the University of Melbourne (Kaye Stacey) since 2008
Cycles of Research-based design with investigating more than 500.000 students' solutions

About 130 Tests (every 5-10 min) in 5 areas & 65 topics
For teachers and their spontaneous practices during teaching

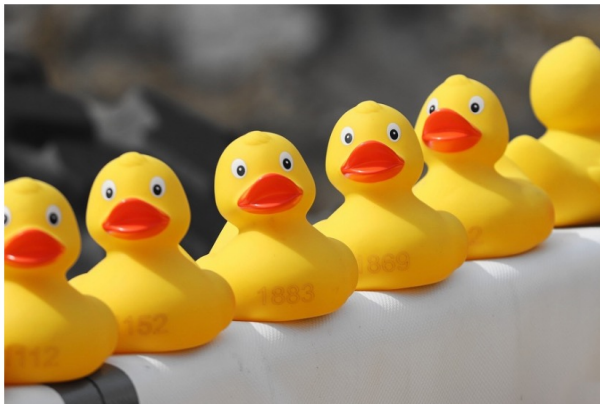


Digital diagnostics in Germany



Illustrative example: Meaning of letters

0%



Lucy hat 6 Enten für insgesamt 12 Euro gekauft.

Sie hat folgende Gleichung aufgeschrieben: $6e = 12$.

Wofür steht das e in Lucys Gleichung? e steht für:

- Enten
- eine Ente
- den Preis einer Ente
- Euro

Seite 1 / 9

WEITER



Student focus & adaptivity:
Work with student perspectives

SMART Deutsches Zentrum für Lehrkräftebildung Mathematik

0%

Payam hat für seinen Garten r rote Rosen-Sträucher und l lila Lavendel-Pflanzen gekauft.

Ein Rosen-Strauch kostet jeweils 4 €. Eine Lavendel-Pflanze kostet jeweils 5 €.

Welche Gleichung gibt an, dass die Pflanzen insgesamt 70 Euro gekostet haben?

$4r + 5l = 70$

$10r + 6l = 70$

$r + l = 70$

Seite 2 / 9

ZURÜCK WEITER

DZLM

SMART Deutsches Zentrum für Lehrkräftebildung Mathematik

0%

Kugelschreiber werden in 3er-Packungen verkauft.

Sam hat p Packungen gekauft und hat jetzt insgesamt k Kugelschreiber.

Wähle die passende Gleichung aus:

$k + p = 4$

$p = 3k$

$p = 3$

$3p = k$

$30k = 10p$

Seite 3 / 9

ZURÜCK WEITER

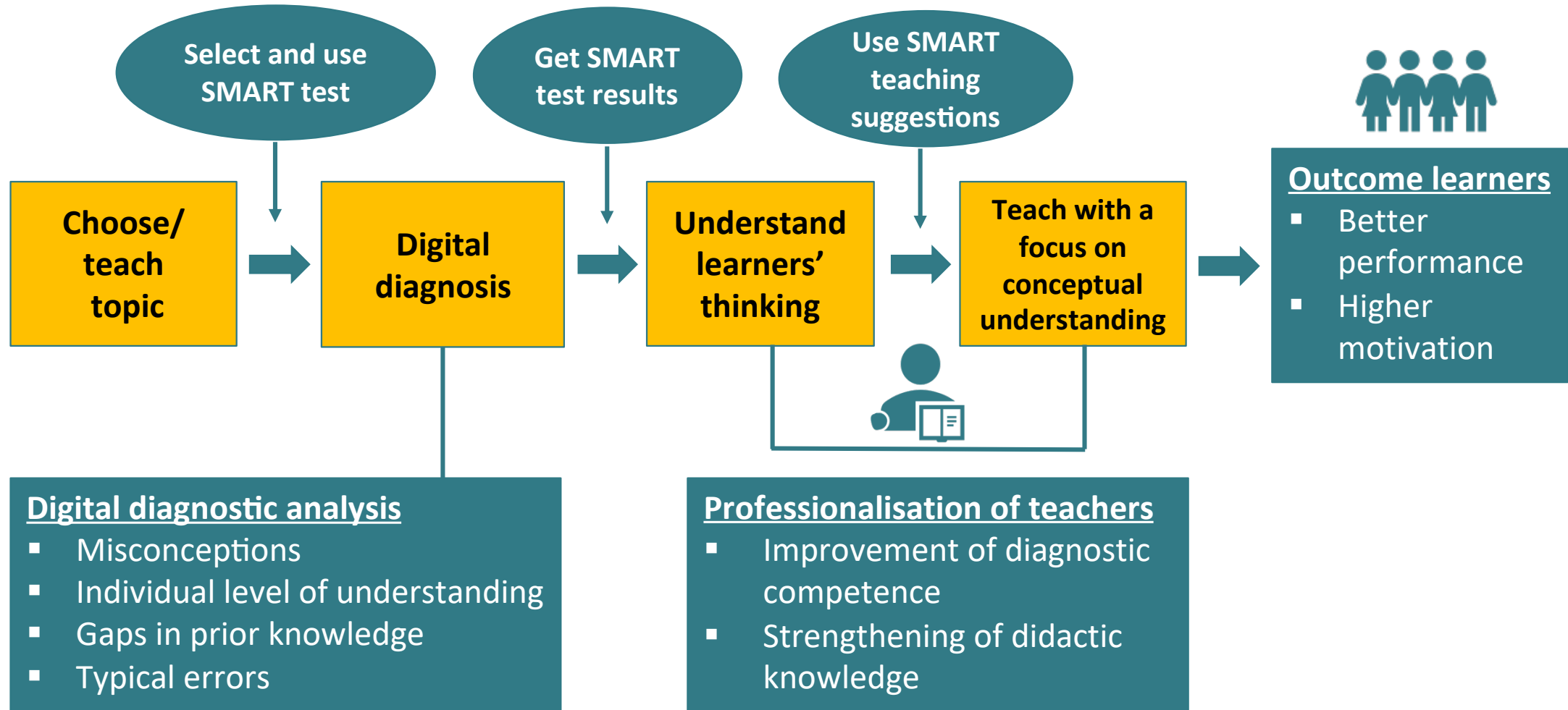
DZLM

Digital diagnostics in Germany



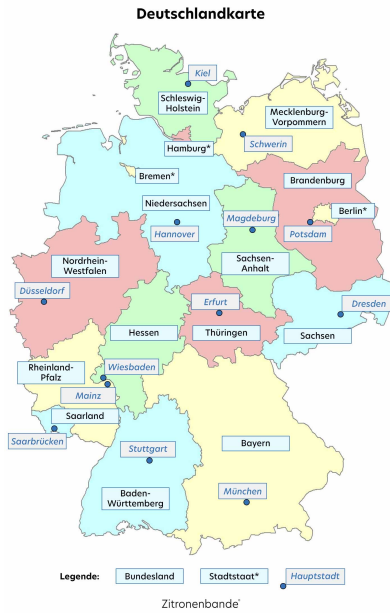
Student focus & adaptivity:
Work with student perspectives

SMART

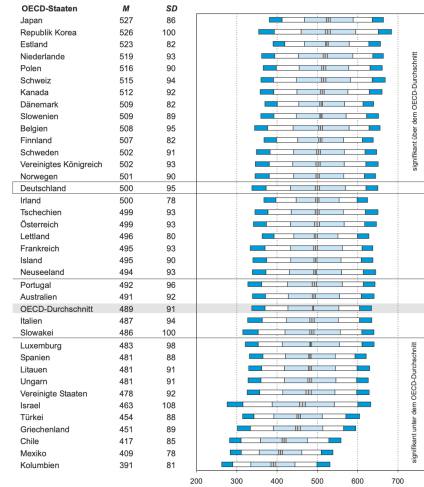


Revised National Standards for Mathematics:

The story



New challenges



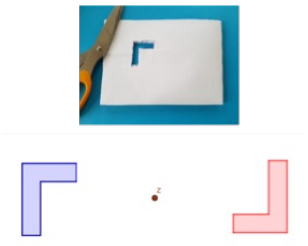
Two thirds of 8th graders in Germany say they never work with media in MU

Illustrative Examples

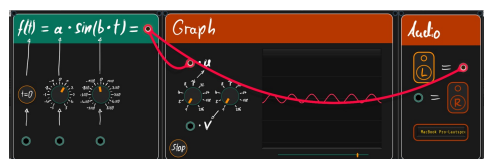
- Cognitive demand:** Initiate active learning processes
- Focus on understanding:** of concepts, strategies and procedures
- Longitudinal coherence:** Prepare for sustainable learning
- Student focus & adaptivity:** Work with student perspectives
- Enhanced communication:** Talk about mathematics

Space and Shape

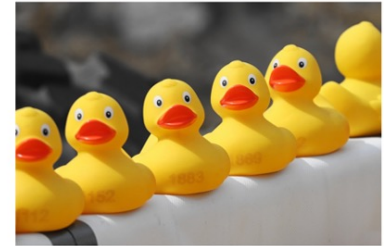
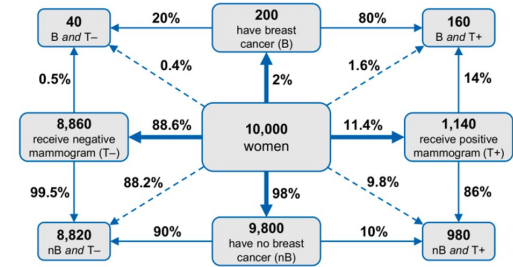
Structures and functional dependency



Multiplikation von Brüchen				
3	*	9	=	27
8	*	5	=	40



Data and Chance



Many thanks four your attention!

Bärbel Barzel, University of Duisburg-Essen

Pictures from IQB



Institut zur Qualitätsentwicklung
im Bildungswesen



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DZLM 



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Offen im Denken

- Literature**
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